Practice Midterm Exam Questions with Solutions

Table One		Table Two	
A	**B**	**C**	**D**
0	2	1	3
1	3	2	6
2	3	3	9
3	4	4	12

1. For each of the given (input, output) relationships in Tables One and Two; answer the following: Is the relationship a function? Why or why not?

   a. A a function of B:
   No, 3 has two different possible outputs.

   b. B a function of A
   Yes, for each input there is only one possible output.

   c. C a function of D
   Yes, for each input there is only one possible output.

   d. D a function of C
   Yes, for each input there is only one possible output.

   For the relationships that are functions; also answer:

   i. What is the domain of the function?

   ii. What is the range of the function?

B is a function of A: A={0,1,2,3} is the domain, B={2,3,4} is the range.
C is a function of D: D={3,6,9,12} is the domain, C={1,2,3,4} is the range.
D is a function of C: C={1,2,3,4} is the domain, D={3,6,9,12} is the range.

2. What is the domain and range for each of the following functions? Show your work for determining the domain when applicable. You may find graphing the function or using the calculator table feature for the function to be helpful; especially for determining the range. If you do so, include a sketch of the graph (label key points).

   a. $f(x) = \dfrac{1}{3x-3}$

   The function is defined for all real numbers where the denominator is not 0. The denominator is 0 when 3x-3=0 or x=1. So the Domain is {x in R| x≠1} or "all Real numbers except 1".

   The range is the set of all values that f(x) can take on. If I set y = f(x), I can solve for x as long as y is not 0 (there is no value of x that will give me 0) so the range is {y in R | y ≠0} or "all real numbers except 0".

   b. $f(x) = \dfrac{1}{1+x^2}$

   The denominator is always positive so it will never be zero, the domain here is all Real numbers. However the range is just positive numbers (y>0) since the output will always be positive (and not 0).

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c. $f(x) = \sqrt{5x - 10}$

The square root is undefined for real numbers that are negative so for the domain I need 5x-10≥0 – solving for x I get the domain is {x in R | x ≥ 2}

The range is {y in R| y≥0}

d. $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$

For this one I need the denominator not zero for the rational function and what is inside the square root to be positive, therefore the domain is when $4-x^2 > 0$ or {x in R | -2< x < 2}. Once again only positive (non-zero) values will occur so the range is {y in R |y >0}

e. $f(x) = \dfrac{1}{\sqrt{4x - 2}}$

As with the reasoning for d), I need 4x-2 >0 or x > ½ in order for f(x) to be defined so the domain is {x in R | x>1/2} and the range is {y in R | y>0}

f. $y = \sqrt{(x - 2)^3}$

I can only take the square root of a value greater than or equal to zero. When I cube a value it is negative if and only if what I'm cubing is negative (i.e. x^3 is negative if and only if x is) so I need x-2≥0 or {x in R | x ≥ 2} is the domain and the range is {y in R |y≥0} (notice I can include 0 in the range when it is just the square root, but not when the square root is in the denominator.

3. Use the provided graph of Eugene's bike trip to answer the following questions: Be sure you can find equations of any of the requested lines.
   a. When is Eugene taking a break? How do you know?
      Eugene is taking a break from 30-50 minutes because his distance from home doesn't change (the distance line is horizontal).

   b. When is Eugene going away from or towards his house? How do you know?
      Eugene is going away from home from 0-30, 50-80, and 90-120 minutes. I know because the distance graph is increasing on those intervals.

      Eugene is going towards home between 80 and 90 minutes. I can tell because the distance graph is decreasing then. It looks like he is going fast - maybe he dropped his candy bar and went back for it.

   c. When is Eugene going faster and faster? Explain how you know using calculus concepts.

      Eugene is going faster and faster from 70-80 (forward), 80-88 (approx) (backward) and 90- approx 98 minutes (forward) because the <u>slopes</u> are getting steeper during those intervals which indicates he is going faster and faster (either toward or away from home)

d. When is Eugene going slower and slower? Explain how you know using calculus concepts.

   Eugene is going slower and slower from approx. 88 to 90 and 98-100. I can tell because the slopes are getting less steep during those times. This is a little hard to see on the graph.

e. When is Eugene going at a constant rate and what are his speeds in mph during those times? Explain how you know using calculus concepts.

   Eugene is going at a constant rate from 0-30 minutes. His speed is the slope of the line which is 5/30 = 1/6 miles/ minute.

f. When is Eugene going the fastest and how fast, in mph is he going? Which calculus concept does this relate to?

   Eugene is going the fastest where the slope is the steepest. To me it looks like this occurs around 87 minutes when he is headed toward home. The calculus concept relates to the maximum of the derivative since the derivative of distance is speed.

g. What is Eugene's average speed, in mph, during the first 45 minutes of his trip? The second 45 minutes? Which calculus concept does this relate to?

   During the first 45 minutes we can find the average by looking at the slope of the secant line connecting the points (0,0) and (45,5). The slope, or average speed, is 5/45 miles/minute.

   During the second 45 I can do the same thing looking at the endpoints (45,5) and (90,5). Even though he has been traveling, the average speed is 0 since the slope of the secant line is zero. This makes sense because he starts and ends in the same place during that second 45 second period.

h. When is Eugene's average speed 9 mph, 12 mph, 15 mph? Which calculus concept does this relate to?
   Since the graph is in miles/minute we should first convert the speeds from mph to miles/minute. 9 miles/hour * 1 hour/ 60 minutes = 9/60=3/20 miles/ minute. So since this is average speed we are looking for two points where the tangent line connecting them has a slope of 3/20. One such pair would be approximately (86,11) and (106, 14) .
   The solutions for 12, and 15 follow similar steps.

i. When is Eugene's speed 9 mph, 12 mph, 15 mph? Which calculus concept does this relate to?

The calculus concept here is the derivative or slope of the tangent line. Like in the last problem we need to convert the speeds to miles/minute. 9mph = 3/20 miles/minute. Looking on the graph it looks like the point (72,10.2) would have a tangent line with that slope. The other problems are done similarly.

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4. Suppose a rock is thrown into the air and its height in feet at time t seconds is given by $h(t)=-16t^2 + 20t + 6$.
   a) What is the initial height of the rock?
   The initial height of the rock is the height at t=0. Plug this into the h(t) function: h(0)=6feet.

   b) When does the rock hit the ground?
   The rock hits the ground when the height is 0. So set h(t)=0 and solve for t. You'll need the quadratic formula here and should get t=1.5 seconds (you also get -0.25, but it doesn't make sense to have negative seconds).

   c) How fast is the rock going when it hits the ground?

The speed of the rock is given by the velocity which is the first derivative: h'(t)=-32t+20. The rock hit the ground at 1.5 seconds so we just plug this in to the velocity function: h'(1.25)=-28 feet/sec.

5. For each of the following, use the Power, Constant Multiple and Sum Rules (as appropriate) to determine $f'(x)$. Carefully show your work, don't skip steps. Give your answers without negative exponents.

a. $f(x)=\dfrac{1}{2x^3}$	b. $f(x)=\sqrt[3]{x^2}$	c. $f(x)=17x^5-22x^3+\pi$
$f'(x)=\dfrac{-3}{2x^4}$	$f'(x)=\dfrac{2}{3\sqrt[3]{x}}$	$f'(x)=85x^4-66x^2$

6. For each of the following functions, sketch the graph of the derivative on the same axes. Carefully mark all key points on the graphs and/or axes and explain your work. Use key terms in your explanations. Label your curves $f(x)$ and $f'(x)$

Key Terms		
	Function	↑, ↓, local max, local min, concave up, concave down
	Derivative	> 0, < 0, = 0, ↑, ↓

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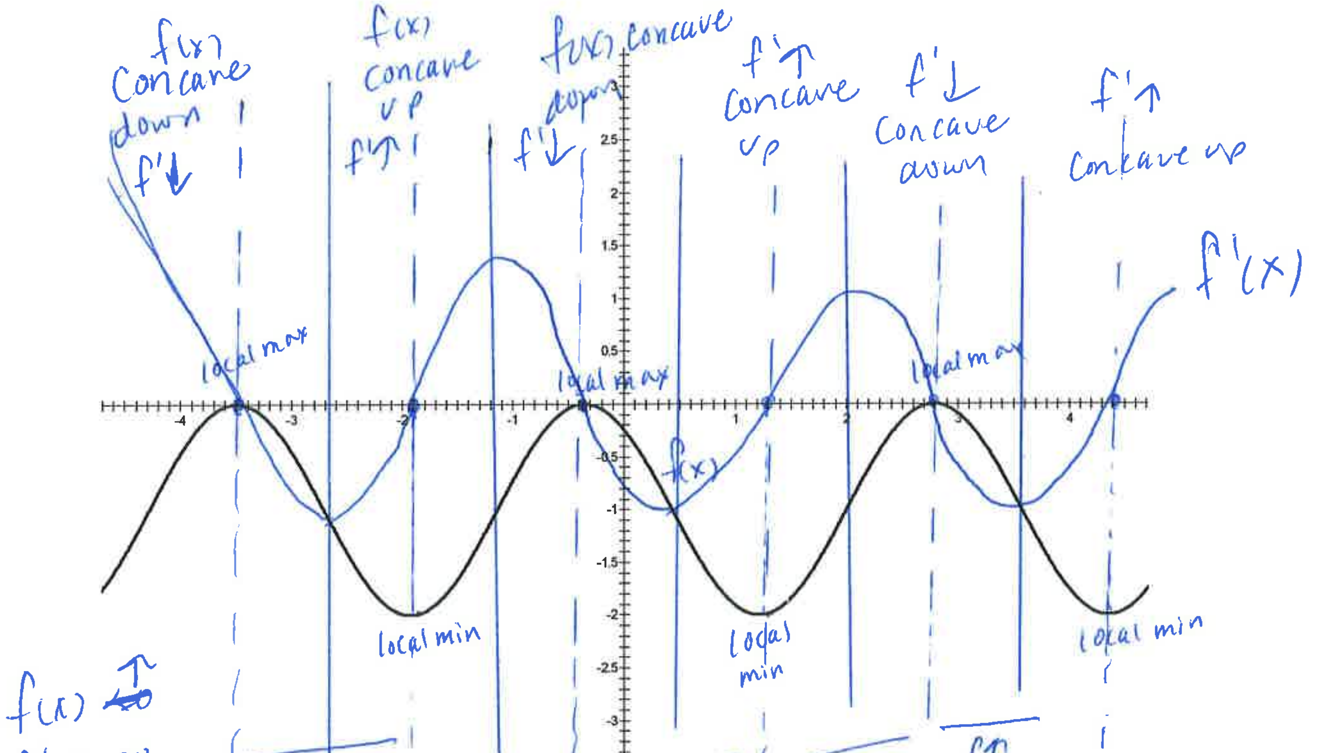

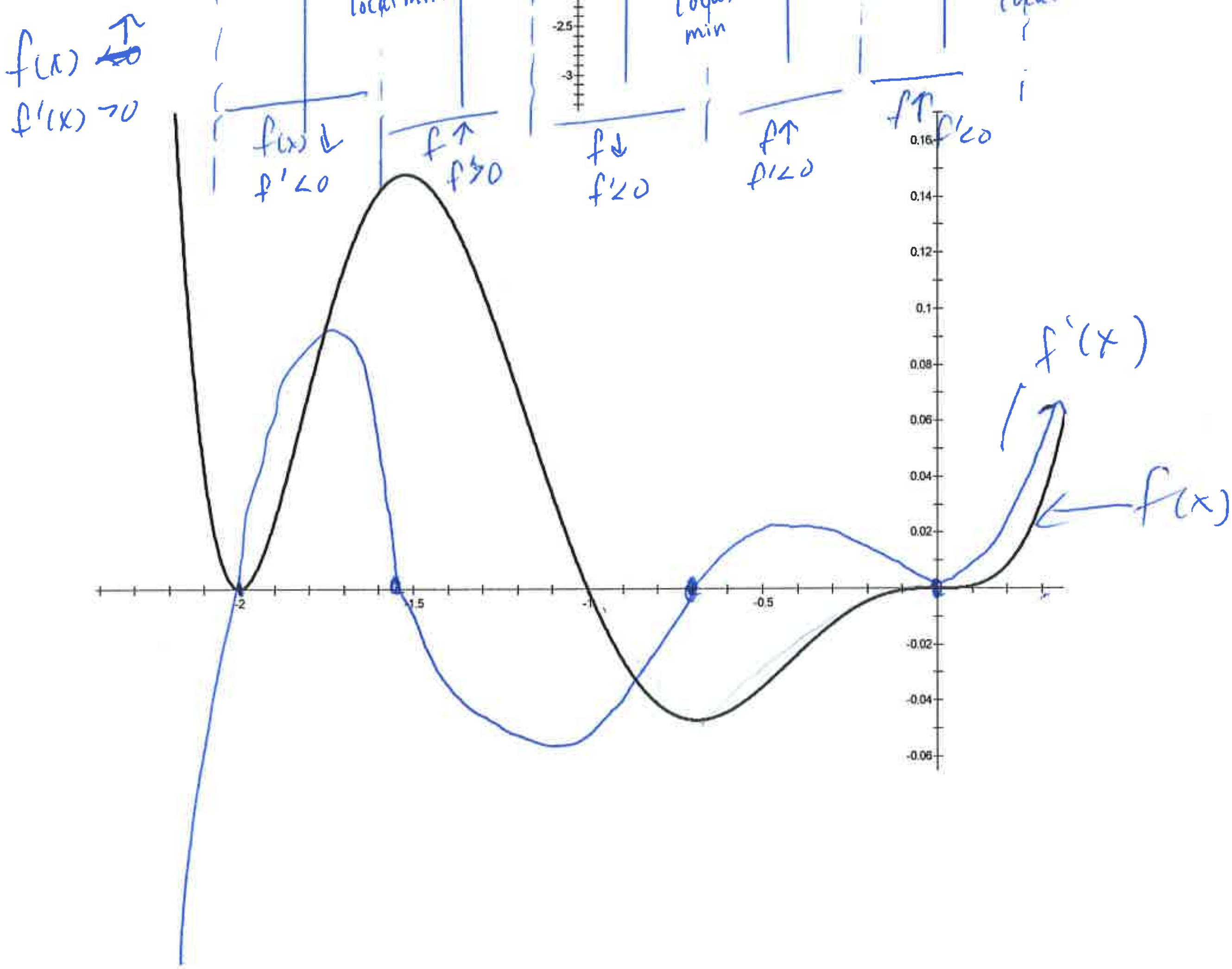

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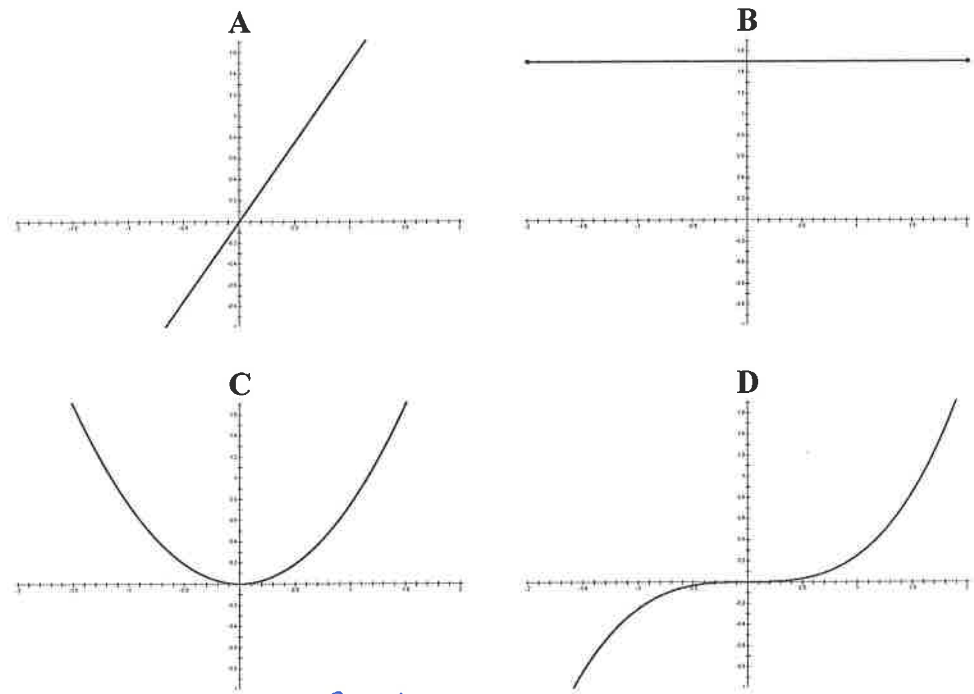
7. Identify as many relationships (graph A is the derivative of graph _____ and the function for graph ___, etc.) as possible. Explain your responses using key terms.

Key Terms

Function	↑, ↓, local max, local min, concave up, concave down
Derivative	> 0, < 0, = 0, ↑, ↓

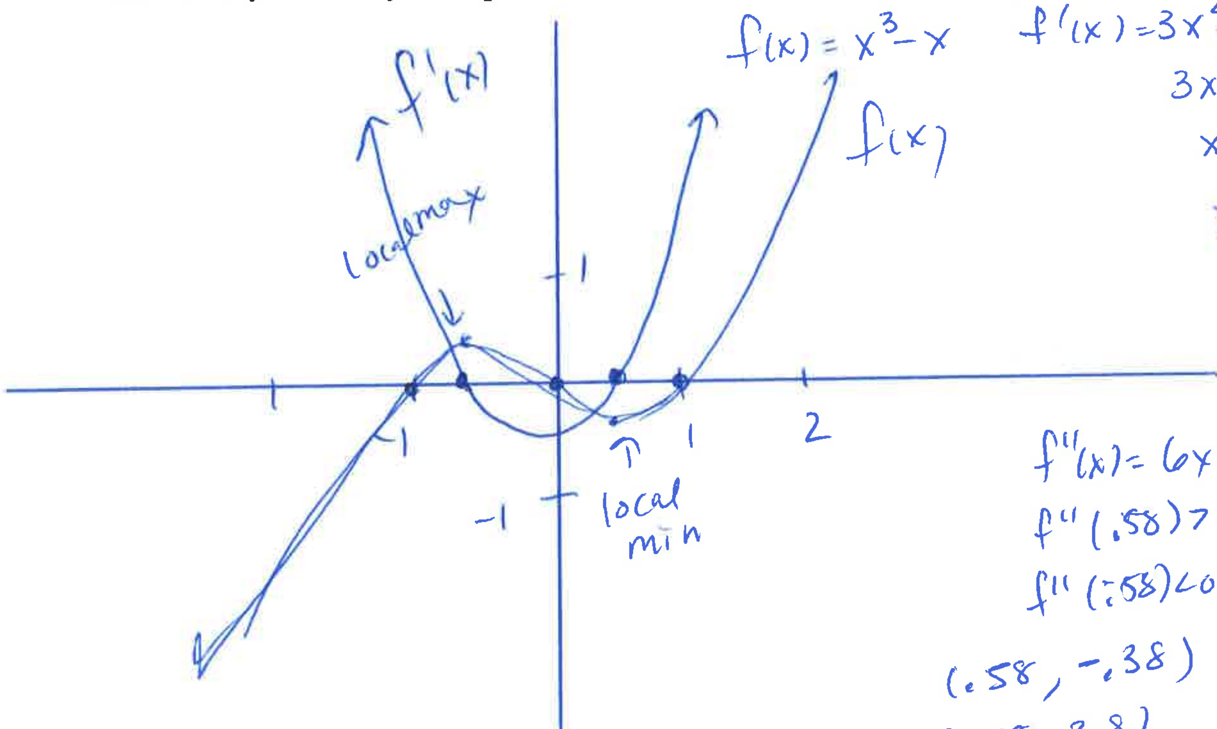


$D' = C$
$C' = A$
$A' = B$

$x(x^2-1)$

8. Carefully sketch $f(x) = x(x+1)(x-1)$ and then sketch the graph of the derivative of $f(x)$ on the same axes. Carefully mark all key points on the graphs and/or axes and explain your work. Use key terms in your explanations. Label your curves $f(x)$ and $f'(x)$

$f(x) = x^3 - x$



$f'(x) = 3x^2 - 1 = 0$

$3x^2 = 1$

$x^2 = \frac{1}{3}$

$x = \pm\sqrt{\frac{1}{3}} \approx \pm .58$

critical pts

$f''(x) = 6x$

$f''(.58) > 0$ concave up

$f''(-.58) < 0$ concave down

$(.58, -.38)$

$(-.58, .38)$