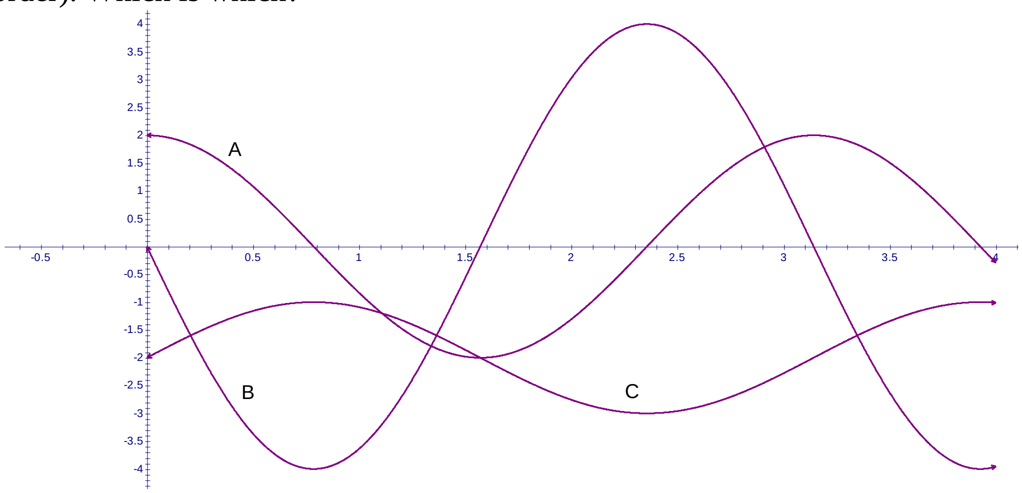
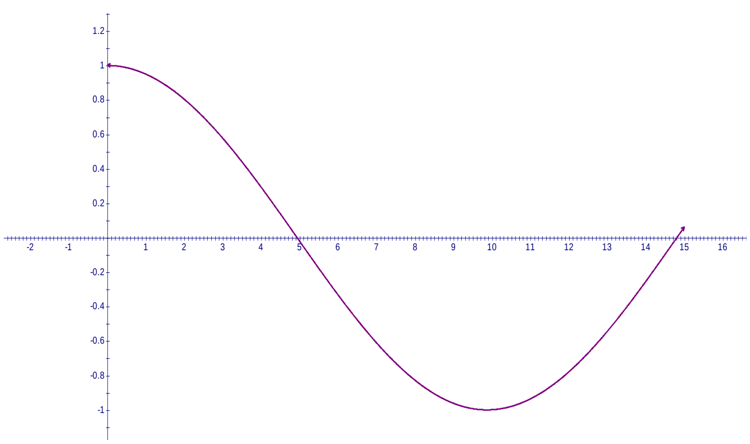
- The Final Exam is Wednesday 6/13 @ 4-5:50 PM in our usual room (MNB 104)
- You may have 2 notecards, calculator and a ruler (rulers and calculators can be borrowed during the exam)
- In addition to reviewing the coursepack and homework questions for the term, the following questions are for review. The answers will be posted on our course webpage sometime over the weekend.

1. The graphs A, B and C are a function and its first and second derivatives (not necessarily in that order). Which is which?



2. Suppose that the graph below represents the *derivative*, $f'(x)$, of some function, $f(x)$.
The axes are a little hard to read, but assume the y-intercept is (0,1) and the x-intercepts are (5,0) and (14.75,0). For each question below answer the question or explain why the answer cannot be determined from the graph of $f'(x)$.



   a. On what interval(s) is $f(x)$ increasing?
   b. On what interval(s) is $f(x)$ concave down?
   c. On what interval(s) is $f(x)$ negative?
   d. Where does $f(x)$ have an inflection point?

e. Where is $f''(x) > 0$?

f. The point $(5,0)$ is a critical point for $f(x)$ (since a critical point is where the derivative is zero). Is $(5,0)$ a local max, min, or neither? Explain.

3. Suppose you are constructing a rectangular fence to enclose 100 square feet of garden. One side of the fence must be wood and costs \$20 per linear foot to construct; the other three sides are wire and cost \$5 per linear foot to construct. Additionally, the side parallel to the wooden side will have shrubs planted along it at a cost of \$10 per linear foot. Use calculus to determine the dimensions and cost (including the shrubs) of the least cost fence.

4. Suppose a rock is hurled vertically up (on Earth, where the acceleration due to gravity is -32 ft/sec^2). Suppose further that the rock is 186 feet above ground at $t = 2$ seconds and the velocity at $t = 3$ seconds is 4 feet / second.

(i) Find the height $(h(t))$, velocity $(v(t))$ and acceleration $(a(t))$ functions

(ii) Completely describe the path of the rock (max height, when did it hit the ground, how fast was it going when it hit the ground)

(iii) Determine the speed of the rock at $t = 1.5$ seconds (find the equation for and graph the corresponding tangent line) and find another time when the rock is going the same speed it was going at $t = 1.5$ seconds (find the equation for and graph the corresponding tangent line).

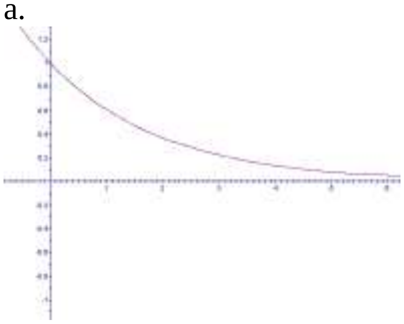
(iv) Find the average speed of the rock over the first three seconds of travel (what calculus concept does this relate to?).

5. Find the derivative of each of the following

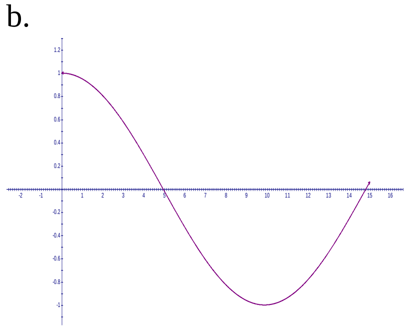
a. $sin(x^3 - 3x)$ b. $y = (1+7x)(3x+4)^{13}$ c. $y = (1+7x^3)^{12}$ d. $y = \dfrac{1+7x^3}{2-x^4}$

6. For each of the following graphs, i) resketch the graph on graph paper, assume the graph is the function and sketch the derivative, ii) resketch the graph on graph paper, assume the graph is the derivative and sketch an antiderivative

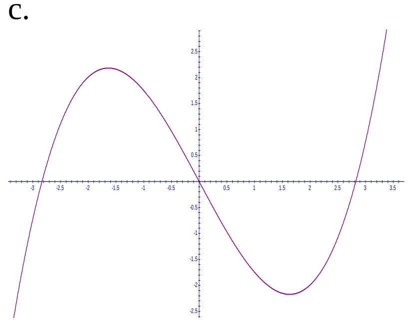
a.



b.



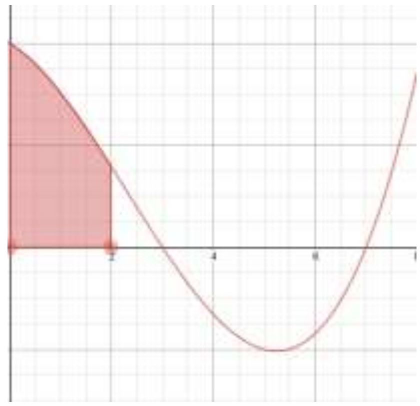
c.



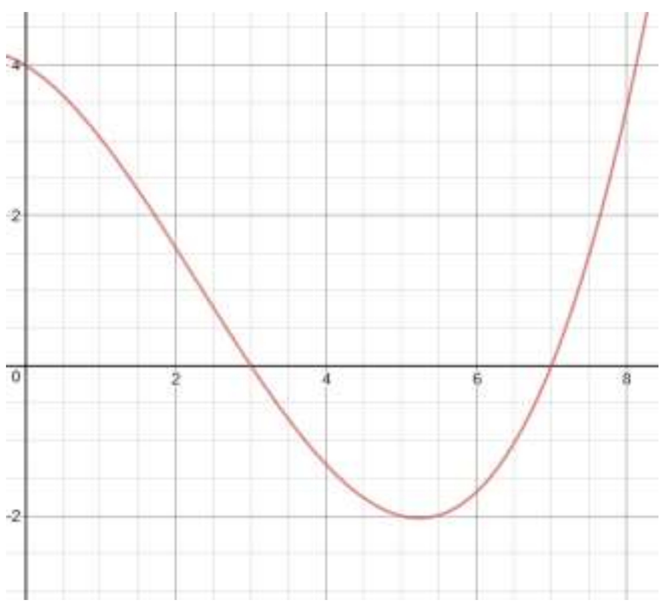
7. Using calculus, compute each of the following:

a. $\int_{-2}^{4} 2x^3 + x \, dx$ b. $\int_{1}^{4} \frac{2}{x^3} \, dx$ c. $\int 2\cos(x) - e^x dx$

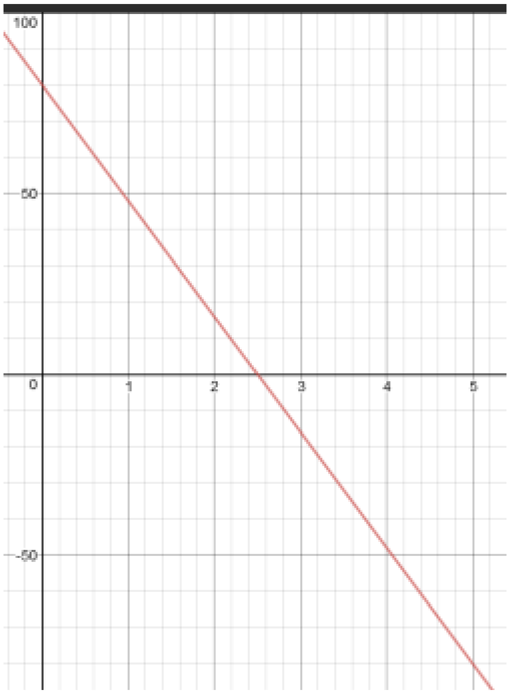
8. Write an integral that represents the indicated signed area under the graph of $f(x)$ below.



9. Estimate the value of $\int_0^4 f(x)dx$.



10. A rock is thrown into the air and the *velocity*, $v(t)$, of the rock is shown in the graph below.



a. What was the initial velocity of the rock?
b. At what time did the rock reach its highest point? How can you tell from the graph?
c. If the rock hit the ground after 5 seconds, then write a definite integral that represents the *total* distance the rock traveled.

11. For each of the given (input, output) relationships in Tables One and Two; answer the following: Is the relationship a function? Why or why not?

a. A a function of B
b. B a function of A
c. C a function of D
d. D a function of C

For the relationships that are functions; also answer:

i. What is the domain of the function?
ii. What is the range of the function?

Table One		**Table Two**	
A	**B**	**C**	**D**
0	2	1	3
1	3	2	6
2	3	3	9
3	4	4	12

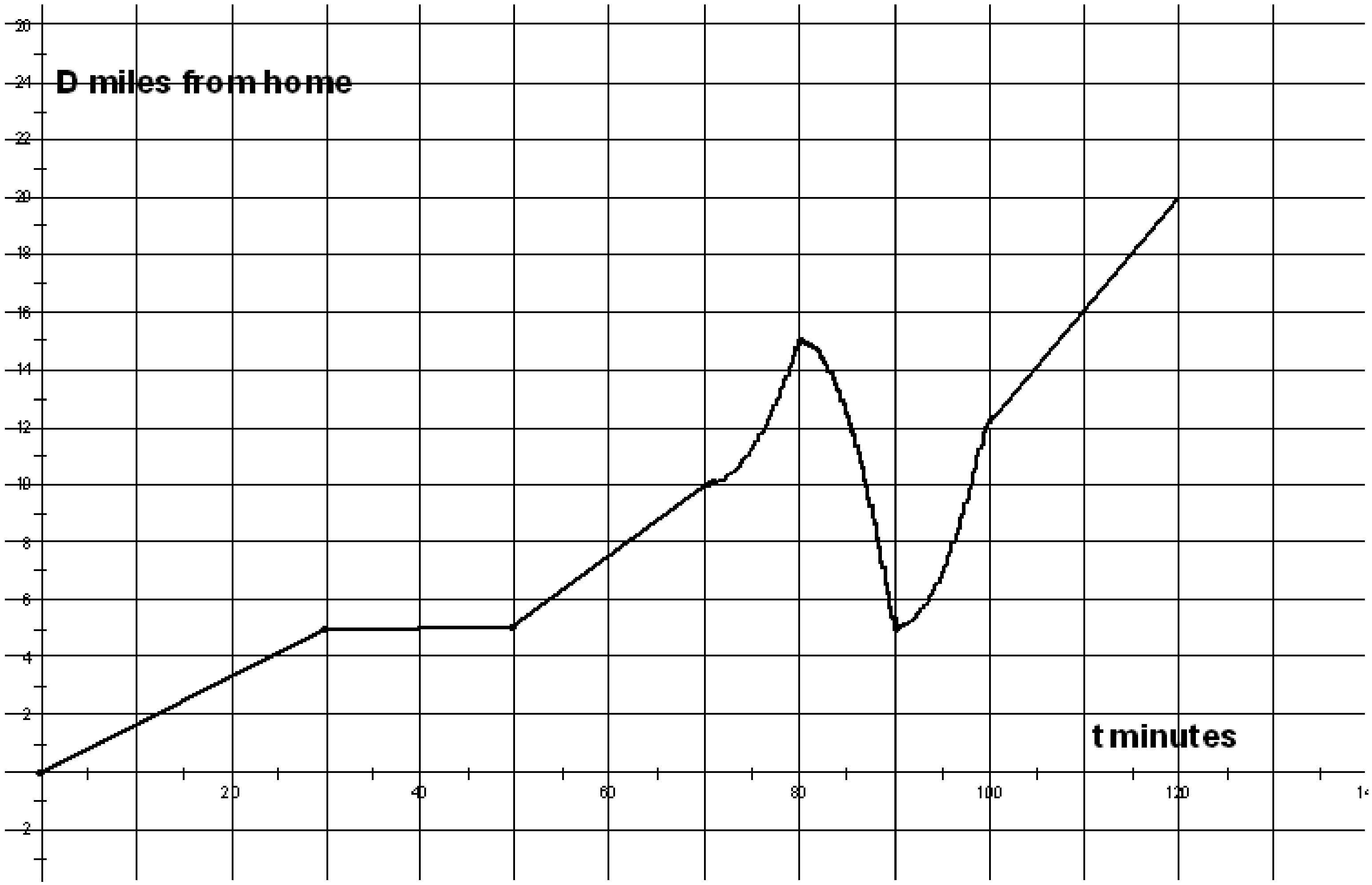
12. What is the domain and range for each of the following functions? Show your work for determining the domain when applicable.

a. $f(x)=\dfrac{1}{3x-3}$ b. $f(x)=\dfrac{1}{1+x^2}$ c. $f(x)=\sqrt{5x-10}$

d. $f(x)=\dfrac{1}{\sqrt{4-x^2}}$ e. $f(x)=\dfrac{1}{\sqrt{4x-2}}$ f. $y=\sqrt{(x-2)^3}$

13. Use the provided graph of Eugene's bike trip to answer the following questions: Be sure you can find equations of any of the requested lines.
    a. When is Eugene taking a break? How do you know?
    b. When is Eugene going away from or towards his house? How do you know?
    c. When is Eugene's speed increasing? Explain how you know using calculus concepts.
    d. When is Eugene going at a constant rate and what are his speeds in mph during those times? Explain how you know using calculus concepts.
    e. When is Eugene going the fastest and how fast, in mph is he going? Which calculus concept does this relate to?
    f. What is Eugene's average speed, in mph, during the first 45 minutes of his trip? The second 45 minutes? Which calculus concept does this relate to?
    g. When is Eugene's average speed 9 mph, 12 mph, 15 mph? Which calculus concept does this relate to?
    h. When is Eugene's (instantaneous) speed 9 mph, 12 mph, 15 mph? Which calculus concept does this relate to?

MATH 495.595 / PRACTICE MIDTERM EUGENE'S BIKE TRIP



Practice Final Exam Questions

14. Use the numerical method (tables with right and left hand limits) to find the derivative of $f(x) = \cos(x)$ at $x = 1.5$. Find answer to nearest thousandth. Be sure your calculator is in radians.

15. Use Fermat's Method to compute the derivatives of $f(x) = 5x^2 + 1$ and $f(x) = 2 - x^3$. Carefully show all of your steps.

16. For each of the following, use the Power, Constant Multiple and Sum Rules (as appropriate) to determine $f'(x)$. Carefully show your work, don't skip steps. Give your answers without negative exponents.

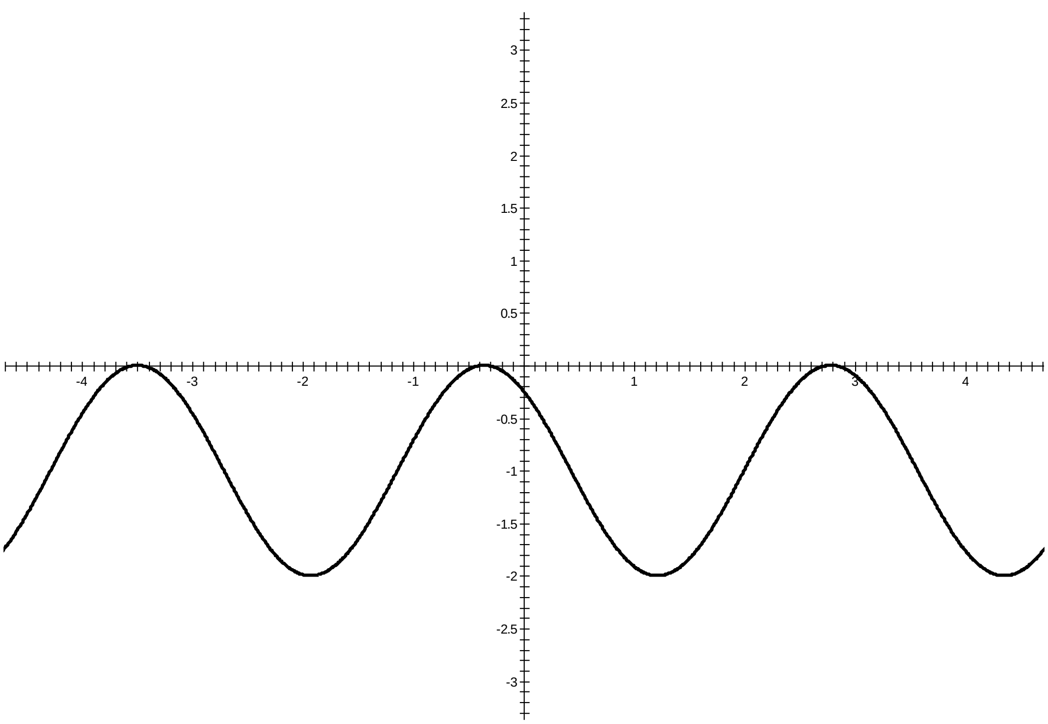
    a. $f(x) = \dfrac{1}{2x^3}$ b. $f(x) = \sqrt[3]{x^2}$ c. $f(x) = 17x^5 - 22x^3 + \pi$

17. Find the equation of the tangent line to $f(x) = 3x^2 - 5x + 1$ at x=2.

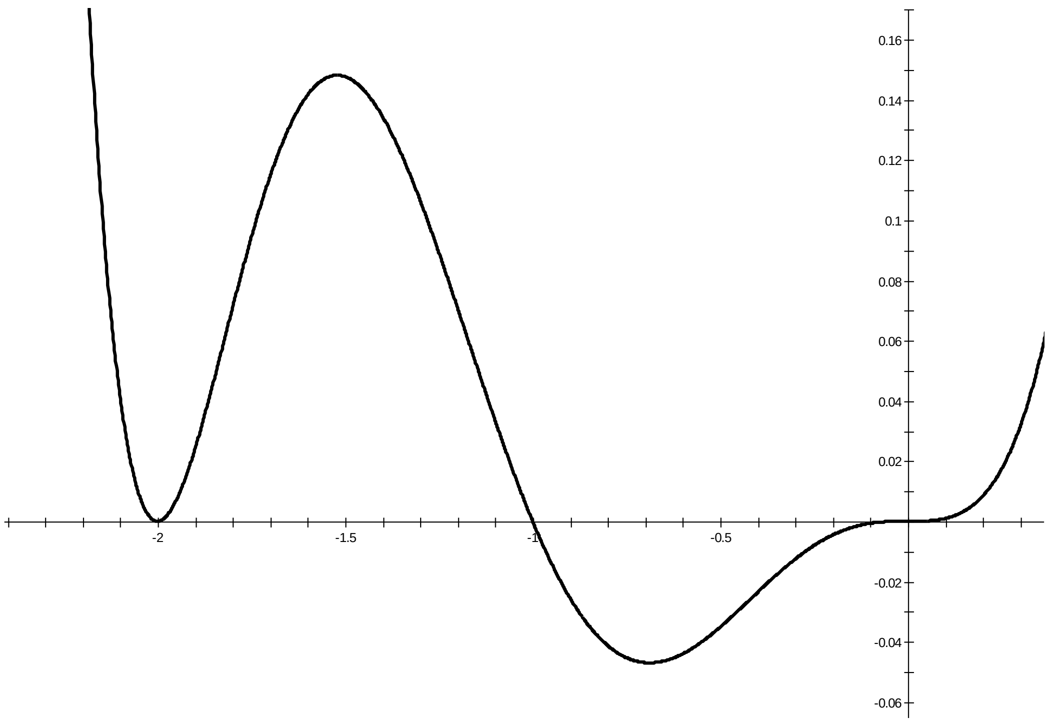
18. For each of the following functions, sketch the graph of the derivative on the same axes. Carefully mark all key points on the graphs and/or axes and explain your work. Use key terms in your explanations. Label your curves $f(x)$ and $f'(x)$

Key Terms

Function	↑ , ↓, local max, local min, concave up, concave down
Derivative	> 0, < 0, = 0, ↑ , ↓



Practice Final Exam Questions



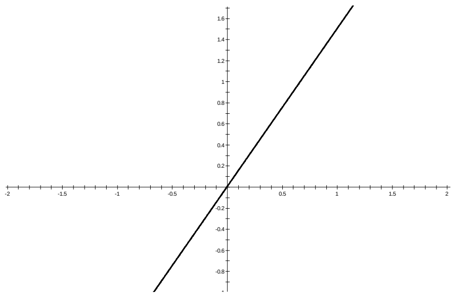
Practice Final Exam Questions

19. Identify as many relationships (graph A is the derivative of graph _____ and the function for graph ___, etc.) as possible. Explain your responses using key terms.

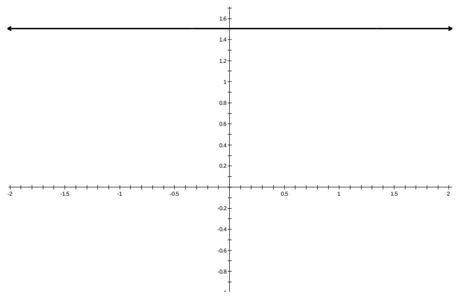
Key Terms

Function	↑ , ↓, local max, local min, concave up, concave down
Derivative	> 0, < 0, = 0, ↑ , ↓

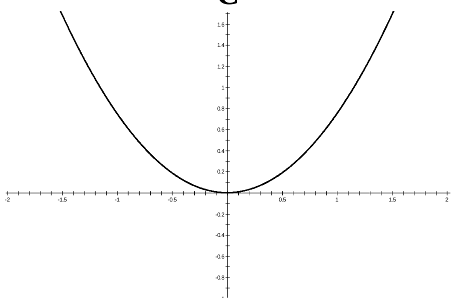
A



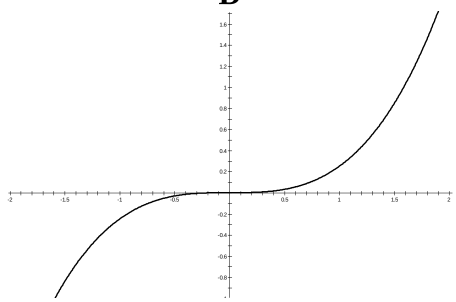
B



C



D



20. Consider $f(x) = \frac{1}{3}x^3 + 2x^2 - 21x$ with a domain of all real numbers.

a. Use the first and second derivatives to find all local maximum and local minimum values.

b. Where does $f(x)$ have an inflection point?

c. On what intervals is $f(x)$ increasing?

d. On what intervals is $f(x)$ decreasing?

e. On what intervals is $f(x)$ concave up?

f. On what intervals is $f(x)$ concave down?

g. Using the above information sketch a graph of $f(x)$.