Math 366: Final Exam Review Sheet

• The Final exam is Thursday, June 9, 2011 from 10-11:50am in MNB 104.
• The Final will cover §4.1 – §4.2; §5.3 – §5.7; §6.2 – §6.6; §7.1 – §7.6.
• You may use a full page of notes (one side only); your calculator; the tables in back of your book; and the front and back covers of your book.
• This is not meant to be an exhaustive set of examples. Study old homework, quizzes, previous exams and previous review sheets for more practice. If you are having trouble with a particular section work problems from the exercises at the end of the section.

1. A random variable X has p.d.f. $f(x) = 3x^2,\ 0 \le x \le 1$.
   (a) Find μ and σ^2.
   (b) A sample of size $n = 12$ is taken from this distribution. Find, approximately, $P(\frac{3}{4} \le \overline{x} \le \frac{4}{5})$.
2. Approximate $P(29.03 \le \overline{x} \le 30.79)$, where $\overline{x}$ is the mean of a random sample of size 25 from a distribution with mean $\mu = 30$ and variance $\sigma^2 = 6$.
3. If $X \sim b(80, 0.3)$, find the approximate value of $P(31 \le x \le 34)$ using
   (a) The normal approximation.
   (b) The binomial distribution.
4. A cake factory produces cakes that have a label weight of 54.2 grams. Assume the distribution of the weights of these cakes is $N(55.3, 25)$. Let $\overline{X}$ equal the sample mean of 90 cakes that are selected and weighed on a particular shift. Find $P(55.0 \le \overline{x} \le 55.7)$.
5. Let X and Y have joint p.d.f. $f(x, y) = 2e^{-x-y},\ 0 \le x \le y < \infty$.
   (a) Find the marginal p.d.f.s of X and Y, $f_1(x)$ and $f_2(y)$.
   (b) Are X and Y independent?
   (c) Find $P(X \le 1/2; Y \le 1)$
6. A random sample of $n = 50$ people were asked how often they check their email daily. It was found that $\overline{x} = 5.6, s_X^2 = 8.2$. Give a 90% confidence interval for μ.
7. A survey was done to check the results of the previous question, and the following data was recorded:

8	6	4	1	3
7	1	5	2	6

   Find a 90% confidence interval giving an upper bound for μ.
8. An Independent random sample of heights of two kinds of trees yielded the following results, $n = 12$, $\overline{x} = 65.7$ feet, $s_x = 4$ feet, $m = 15$, $\overline{y} = 68.2$ feet, $s_y = 3$ feet. Find an approx 95% CI for the difference $\mu_X - \mu_Y$ of the means of the height, assume $\sigma_X^2 = \sigma_y^2$.
9. Students weighed (in kilos) at the beginning and end of a semester-long health fitness program. Let the random variable D equal the weight change for a student, postweight minus preweight. Assume the distribution of D is $N(\mu_0, \sigma_0^2)$. A random sample of $n = 12$ female students yielded the following observations of D:

2.0	−0.5	1.4	−2.2	0.3	−0.8
3.7	−0.1	0.6	0.2	0.9	−0.1

   (a) Give a point estimate of μ_0.
   (b) Find a 95% CI for μ_0.
   (c) What can you conclude?

10. Does the number of cups of tea effect the number of hours of sleep at night? Use $\alpha = 0.05$.

Cups	Hours	of	sleep
0	7	8	5
1	6	7	6.5
2	9	8	8
3	9	7	7

11. Does your age have an effect on the number of hours of sleep you get at night? Use $\alpha = 0.05$ and find the p-value (approximate) of this test.

Age	Hours	of	sleep
< 10	9	10	10.5
$10 - 15$	8	9	8.5
$16 - 24$	6	7	7
$25 - 35$	7	7	8

12. Five different styles of clay pots are tested for their strength when cooked at 3 different heats. The table for their strengths is below:

Heats	A_1	A_2	A_3	A_4	A_5
B_1	44	47	42	51	52
B_2	49	55	43	52	60
B_3	45	48	38	44	56

(a) Use a 5% significance level to test $H_B : \beta_1 = \beta_2 = \beta_3 = 0$ against all alternatives.

(b) Use a 5% significance level to test $H_A : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$ against all alternatives.

13. Assume that the number of doughnuts eaten by students during a statistics class has a normal distribution with $\mu = 1.5$ and $\sigma^2 = 1.25$. How many doughnuts should the teacher bring so that the probability of running out of doughnuts is less than 10% in a class of 23 students.

14. Suppose X_1 and X_2 are independent random variables with distributions $N(10, 25)$ and $N(15, 30)$ respectively. Let $Y = X_1 - X_2$.

(a) What is the mean and variance of Y?

(b) Find $P(X_1 < 12; X_2 < 13)$

15. There were 100 independent trials of an experiment run to test $H_0 : \mu = 50$ against $H_1 : \mu > 50$. The critical region was $C = \{\overline{x} : \overline{x} \geq 50.2\}$. Assume that $\sigma^2 = 4$.

(a) What is the significance level, α of this test?

(b) What sample size would be needed if you wanted to make $\alpha = 0.05$? (keeping the same critical region)

(c) If instead of changing the sample size, we wanted to change the critical region to make $\alpha = 0.05$ (leave $n = 100$) then what would be the new critical region?

16. A confidence interval for the mean of a normal random variable was given that had total length 5, confidence level $\alpha = 0.025$ and the standard deviation was $\sigma^2 = 100$. How many samples did the researchers use?

Answer the following True or False

17. A 90% confidence interval for μ means that 90% of the intervals thus constructed should contain μ.

18. A 90% CI for $\mu_X - \mu_Y$ is $(-10.13, -0.27)$. We can conclude that μ_X is smaller than μ_Y.

19. The $p-$value associated with a test is the probability under H_1 (the alternative hypothesis) that the test statistic is equal to or exceeds the observed value in the direction of H_0 (the null hypothesis).

20. A type I error is defined as the probability of rejecting H_0 and accepting H_1 when the null hypothesis is true.

21. Using an ANOVA test with $m = 5$ and $n = 35$ and testing $H_0 : \mu_1 = \mu_2 = \cdots = \mu_5$, at an $\alpha = 0.01$ significance level, H_0 is accepted if the computed $F = \dfrac{SS(T)/(5-1)}{SS(E)/(35-5)} \geq 4.02 = F_{0.01}(4, 30)$.

22. $\displaystyle\sum_{i=1}^{m}\sum_{j=1}^{n_i}(X_{ij} - \overline{X}_{i.})^2 = SS(TO) - SS(T)$.

Short Answer

23. The Central Limit Theorem is used to approximate a distribution as if it were $\sim N(\_\_, \_\_)$.

24. The Normal is a good approximation to the Binomial if $np \geq$ _____?

25. As the sample size, n, increases and α remains the same, the confidence interval gets (larger / smaller).

26. A 95% CI for $\mu_X - \mu_Y$ is $(-3.23, 0.02)$. We can conclude that _____.

27. What table should you use to test the mean if σ^2 is unknown?

Answers

1. (a) $\mu = \frac{3}{4}$; $\sigma^2 = \frac{3}{80}$.
   (b) 0.3133
2. 0.9224
3. (a) 0.0507
   (b) 0.0524
4. 0.4921
5. (a) $f_1(x) = 2e^{-2x},\ \ 0 \le x \le \infty$; $f_2(y) = 2e^{-y} - 2e^{-2y},\ \ 0 \le y \le \infty$
   (b) no
   (c) $2e^{-3/2} - 3e^{-1} + 1 = 0.3426$
6. $(4.93, 6.27)$
7. $(0, 5.39)$
8. $(-5.273, 0.273)$
9. (a) 0.45
   (b) $(-0.49, 1.39)$
   (c) Since O is in the confidence interval, it is inconclusive whether the program helped students lose weight.
10. $F = 2.11$, $F_{0.05}(3, 8) = 4.09$, do not reject H_0 (number of cups does not appear to effect hours of sleep)
11. $F = 15.48$, $F_{0.05}(3, 8) = 4.07$, Reject H_0 (your age does have an effect on the number of hours of sleep)
12. (a) $F_B = 15.5$ - reject H_B (there is a column effect - the type of pot matters)
    (b) $F_A = 7.62$ - reject H_A (there is a row effect - the heat matters)
13. 42
14. (a) $\mu_Y = -5; \sigma^2 = 55$
    (b) 0.2331
15. (a) $\alpha = 0.1587$
    (b) $n = 271$
    (c) $C = \{\overline{x} : \overline{x} \ge 50.329\}$
16. 81

TRUE/FALSE

17. TRUE
18. TRUE
19. FALSE
20. TRUE
21. FALSE

22. TRUE

Short answer

23. $N(0, 1)$
24. 5
25. smaller
26. Nothing (since 0 is in the CI)
27. t-table