Math 366: Final Exam Review Sheet

- The Final exam is Thursday, June 9, 2011 from 10-11:50am in MNB 104.
- The Final will cover §4.1 – §4.2; §5.3 – §5.7; §6.2 – §6.6; §7.1 – §7.6.
- You may use a full page of notes (one side only); your calculator; the tables in back of your book; and the
  front and back covers of your book.
- This is not meant to be an exhaustive set of examples. Study old homework, quizzes, previous exams and
  previous review sheets for more practice. If you are having trouble with a particular section work problems
  from the exercises at the end of the section.

1. A random variable \( X \) has p.d.f. \( f(x) = 3x^2, \ 0 \leq x \leq 1 \).
   (a) Find \( \mu \) and \( \sigma^2 \).
   (b) A sample of size \( n = 12 \) is taken from this distribution. Find, approximately, \( P(\frac{3}{4} \leq x \leq \frac{4}{5}) \).

2. Approximate \( P(29.03 \leq x \leq 30.79) \), where \( x \) is the mean of a random sample of size 25 from a
   distribution with mean \( \mu = 30 \) and variance \( \sigma^2 = 6 \).

3. If \( X \sim b(80, 0.3) \), find the approximate value of \( P(31 \leq x \leq 34) \) using
   (a) The normal approximation.
   (b) The binomial distribution.

4. A cake factory produces cakes that have a label weight of 54.2 grams. Assume the distribution of the
   weights of these cakes is \( N(55.3, 25) \). Let \( \bar{X} \) equal the sample mean of 90 cakes that are selected and
   weighed on a particular shift. Find \( P(55.0 \leq \bar{X} \leq 55.7) \).

5. Let \( X \) and \( Y \) have joint p.d.f. \( f(x, y) = 2e^{-x-y}, \ 0 \leq x \leq y < \infty \).
   (a) Find the marginal p.d.f.s of \( X \) and \( Y \), \( f_1(x) \) and \( f_2(y) \).
   (b) Are \( X \) and \( Y \) independent?
   (c) Find \( P(X \leq 1/2; Y \leq 1) \).

6. A random sample of \( n = 50 \) people were asked how often they check their email daily. It was found
   that \( \bar{x} = 5.6, s_x^2 = 8.2 \). Give a 90% confidence interval for \( \mu \).

7. A survey was done to check the results of the previous question, and the following data was recorded:
   \[ \begin{array}{cccc}
   8 & 6 & 4 & 1 \\
   3 & 7 & 1 & 5 \\
   2 & 6 & \end{array} \]
   Find a 90% confidence interval giving an upper bound for \( \mu \).

8. An Independent random sample of heights of two kinds of trees yielded the following results, \( n = 12 \),
   \( \bar{x} = 65.7 \) feet, \( s_x = 4 \) feet, \( m = 15, \bar{y} = 68.2 \) feet, \( s_y = 3 \) feet. Find an approx 95% CI for the difference
   \( \mu_X - \mu_Y \) of the means of the height, assume \( \sigma_X^2 = \sigma_Y^2 \).

9. Students weighed (in kilos) at the beginning and end of a semester-long health fitness program. Let
   the random variable \( D \) equal the weight change for a student, postweight minus preweight. Assume
   the distribution of \( D \) is \( \bar{D} \sim N(\mu_0, \sigma_0^2) \). A random sample of \( n = 12 \) female students yielded the following
   observations of \( D \):
   \[ \begin{array}{cccc}
   2.0 & -0.5 & 1.4 & -2.2 \\
   0.3 & -0.8 & 3.7 & -0.1 \\
   -0.1 & 0.6 & 0.2 & 0.9 \\
   -0.1 & \end{array} \]
   (a) Give a point estimate of \( \mu_0 \).
   (b) Find a 95% CI for \( \mu_0 \).
   (c) What can you conclude?
10. Does the number of cups of tea effect the number of hours of sleep at night? Use $\alpha = 0.05$.

<table>
<thead>
<tr>
<th>Cups</th>
<th>Hours of sleep</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7 8 5</td>
</tr>
<tr>
<td>1</td>
<td>6 7 6.5</td>
</tr>
<tr>
<td>2</td>
<td>9 8 8</td>
</tr>
<tr>
<td>3</td>
<td>9 7 7</td>
</tr>
</tbody>
</table>

11. Does your age have an effect on the number of hours of sleep you get at night? Use $\alpha = 0.05$ and find the p-value (approximate) of this test.

<table>
<thead>
<tr>
<th>Age</th>
<th>Hours of sleep</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10</td>
<td>9 10 10.5</td>
</tr>
<tr>
<td>10 − 15</td>
<td>8 9 8.5</td>
</tr>
<tr>
<td>16 − 24</td>
<td>6 7 7</td>
</tr>
<tr>
<td>25 − 35</td>
<td>7 7 8</td>
</tr>
</tbody>
</table>

12. Five different styles of clay pots are tested for their strength when cooked at 3 different heats. The table for their strengths is below:

<table>
<thead>
<tr>
<th>Heats</th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_1$</td>
<td>44</td>
<td>47</td>
<td>42</td>
<td>51</td>
<td>52</td>
</tr>
<tr>
<td>$B_2$</td>
<td>49</td>
<td>55</td>
<td>43</td>
<td>52</td>
<td>60</td>
</tr>
<tr>
<td>$B_3$</td>
<td>45</td>
<td>48</td>
<td>38</td>
<td>44</td>
<td>56</td>
</tr>
</tbody>
</table>

(a) Use a 5% significance level to test $H_B : \beta_1 = \beta_2 = \beta_3 = 0$ against all alternatives.

(b) Use a 5% significance level to test $H_A : \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \alpha_5 = 0$ against all alternatives.

13. Assume that the number of doughnuts eaten by students during a statistics class has a normal distribution with $\mu = 1.5$ and $\sigma^2 = 1.25$. How many doughnuts should the teacher bring so that the probability of running out of doughnuts is less than 10% in a class of 23 students.

14. Suppose $X_1$ and $X_2$ are independent random variables with distributions $N(10, 25)$ and $N(15, 30)$ respectively. Let $Y = X_1 - X_2$.

(a) What is the mean and variance of $Y$?

(b) Find $P(X_1 < 12; X_2 < 13)$

15. There were 100 independent trials of an experiment run to test $H_0 : \mu = 50$ against $H_1 : \mu > 50$. The critical region was $C = \{ \bar{x} : \bar{x} \geq 50.2 \}$. Assume that $\sigma^2 = 4$.

(a) What is the significance level, $\alpha$ of this test?

(b) What sample size would be needed if you wanted to make $\alpha = 0.05$? (keeping the same critical region)

(c) If instead of changing the sample size, we wanted to change the critical region to make $\alpha = 0.05$ (leave $n = 100$) then what would be the new critical region?

16. A confidence interval for the mean of a normal random variable was given that had total length 5, confidence level $\alpha = 0.025$ and the standard deviation was $\sigma^2 = 100$. How many samples did the researchers use?

17. A 90% confidence interval for $\mu$ means that 90% of the intervals thus constructed should contain $\mu$.

18. A 90% CI for $\mu_X - \mu_Y$ is $(−10.13, −0.27)$. We can conclude that $\mu_X$ is smaller than $\mu_Y$. 

Answer the following True or False
19. The $p$-value associated with a test is the probability under $H_1$ (the alternative hypothesis) that the test statistic is equal to or exceeds the observed value in the direction of $H_0$ (the null hypothesis).

20. A type $I$ error is defined as the probability of rejecting $H_0$ and accepting $H_1$ when the null hypothesis is true.

21. Using an ANOVA test with $m = 5$ and $n = 35$ and testing $H_0 : \mu_1 = \mu_2 = \cdots = \mu_5$, at an $\alpha = 0.01$ significance level, $H_0$ is accepted if the computed $F = \frac{SS(T)/(5 - 1)}{SS(E)/(35 - 5)} \geq 4.02 = F_{0.01}(4, 30)$.

22. $\sum_{i=1}^{m} \sum_{j=1}^{n_i} (X_{ij} - \overline{X}_i)^2 = SS(\text{TO}) - SS(T)$.

Short Answer

23. The Central Limit Theorem is used to approximate a distribution as if it were $\sim N(\_, \_)$.

24. The Normal is a good approximation to the Binomial if $np \geq \_\_\_$?

25. As the sample size, $n$, increases and $\alpha$ remains the same, the confidence interval gets (larger / smaller).

26. A 95% CI for $\mu_X - \mu_Y$ is $(-3.23, 0.02)$. We can conclude that $\_\_\_\_\_\_$.

27. What table should you use to test the mean if $\sigma^2$ is unknown?
Answers

1. (a) $\mu = \frac{3}{4}$; $\sigma^2 = \frac{3}{80}$.
   (b) 0.3133

2. 0.9224

3. (a) 0.0507
   (b) 0.0524

4. 0.4921

5. (a) $f_1(x) = 2e^{-2x}$, $0 \leq x \leq \infty$; $f_2(y) = 2e^{-y} - 2e^{-2y}$, $0 \leq y \leq \infty$
   (b) no
   (c) $2e^{-3/2} - 3e^{-1} + 1 = 0.3426$

6. (4.93, 6.27)

7. (0, 5.39)

8. (−5.273, 0.273)

9. (a) 0.45
   (b) (−0.49, 1.39)
   (c) Since $O$ is in the confidence interval, it is inconclusive whether the program helped students lose weight.

10. $F = 2.11$, $F_{0.05}(3, 8) = 4.09$, do not reject $H_0$ (number of cups does not appear to effect hours of sleep)

11. $F = 15.48$, $F_{0.05}(3, 8) = 4.07$, Reject $H_0$ (your age does have an effect on the number of hours of sleep)

12. (a) $F_B = 15.5$ - reject $H_B$ (there is a column effect - the type of pot matters)
   (b) $F_A = 7.62$ - reject $H_A$ (there is a row effect - the heat matters)

13. 42

14. (a) $\mu_Y = -5$; $\sigma^2 = 55$
   (b) 0.2331

15. (a) $\alpha = 0.1587$
   (b) $n = 271$
   (c) $C = \{x : x \geq 50.329\}$

16. 81

   TRUE/FALSE

17. TRUE

18. TRUE

19. FALSE

20. TRUE

21. FALSE
22. TRUE

   Short answer

23. \( N(0, 1) \)

24. 5

25. smaller

26. Nothing (since 0 is in the CI)

27. t-table