Math 366: Exam 2 Review Sheet

The exam will cover §6.1-6.6 and §7.1-7.3. You will be allowed to use the tables in the back of your book as well as your front and back covers and one 3x5 notecard. Disclaimer: This is not meant to be an exhaustive set of examples.

1. A new method of teaching reading comprehension using a special computer game was developed and tested on 4-th graders. Let X denote the score on a standardized exam of students taught using the computer game. Assume that X is $N(\mu, 14)$ and test the hypothesis that $H_0 : \mu_x = 75$ against the alternative hypothesis $H_1 : \mu_x > 75$ with $\alpha = .05$.

   (a) If in a sample of $n = 100$ 4-th graders, $\overline{x} = 75.7$, what is your conclusion?

   (b) What is the p-value of the test?

   (c) Find a one-sided 95% confidence interval giving a lower bound on μ_X. Is 75 in this interval?

   (d) Explain why your answers in a)-c) are consistent.

2. Let X, Y denote the amount of bacteria on the bathroom counters and kitchen counters respectively of college dorms measured in number of bacteria colonies per square foot. In a study of 6 bathrooms and 11 kitchens from various college dorms, the following data was gathered:

   X (bathroom): $10.8, 7.2, 6.1, 12.0, 9.8, 9.1$ ($\overline{x} = 9.16$; $s_x^2 = 4.87$)
   Y (kitchen): $12.1, 7.3, 4.9, 7.6, 11.0, 9.6, 12.7, 8.2, 8.5, 10.9, 7.2$ ($\overline{y} = 9.09$; $s_y^2 = 5.70$)

   (a) Suppose that X is $\mathrm{N}(\mu_x, \sigma_x^2)$ and Y is $\mathrm{N}(\mu_Y, \sigma_Y^2)$ and assume equal variances. State a null and alternative hypothesis comparing μ_X and μ_Y, the mean number of bacteria colonies on the bathroom and kitchen counters.

   (b) Define a test statistic and critical region using $\alpha = .05$.

   (c) Evaluate the test statistic and state your conclusion.

3. Let p_m and p_f denote the percentage of male and females respectively who drink coffee in the morning. Test

   $$H_0 : p_m = p_f, \text{ against } H_1 : p_m < p_f$$

   if in a survey of 150 males, 83 drank coffee in the morning and in a survey of 125 females, 79 drank coffee in the morning. Use $\alpha = .05$ and be sure to state your conclusion.

4. Suppose, X is $N(\mu_x, 100)$ We wish to test the hypothesis:

   $$H_0 : \mu_X = 50$$

   $$H_1 : \mu_X > 50$$

   We take a sample of $n = 75$ and compute $\overline{X}$ to estimate μ_X. Define the critical region as $C = \{\overline{x} \mid \overline{x} \geq k\}$.

   (a) What value should k be if you want a significance level of $\alpha = 0.05$?

   (b) What value should k be if you want the probability of a type II error to be $\beta = 0.05$ when $\mu_X = 60$.

   (c) What is the $p-$value of the test when $\overline{x} = 52.5$?

5. A certain medication is supposed to contain 20% of the active ingredient. As a quality control measure, a sample of 12 was taken and the amount of active ingredient measured. The sample mean was $\overline{x} = 20$ and the sample variance was $s^2 = .1$. The company claims the true mean lies in the interval $(19.836, 20.164)$. What level of confidence is there in this interval?

6. In 2000, a poll showed that 79% of children entering Kindergarten had up-to-date immunizations. To update the estimate, how large a sample size is required to be 98% confident that the new estimate has an error no more that $\pm .05$?

7. (a) A 2005 survey showed that 37% of the American viewing audience watched NBC nightly news. If 2000 people were surveyed, find a 90% confidence interval for the proportion of the American viewing audience watching NBC nightly news.

   (b) If NBC wants to repeat the survey to confirm that the percentage still holds, what sample size should they use to maintain the 90% confidence level and have the error be no bigger that $\epsilon = .02$?

8. Thirteen tons of cheese is stored in some old gypsum mines, including "22-pound" wheels (label weight). A random sample of $n = 9$ of these wheels yielded the following weights in pounds:

21.50	18.95	18.55	19.40	19.15
22.35	22.90	22.20	23.10	

   Assuming that the distribution of the weights of the wheels of cheese is $N(\mu, \sigma^2)$, find a 95% confidence interval for μ.

9. In developing countries in Africa and the Americas, let p_1 and p_2 be the respective proportions of women with nutritional anemia. Find an approximate 90% confidence interval for $p_1 - p_2$, given that a random sample of $n_1 = 2100$ African women yielded $y_1 = 840$ with nutritional anemia and a random sample of $n_2 = 1900$ women from the Americas yielded $y_2 = 323$ women with nutritional anemia. What can you conclude?

10. Researchers are attempting to find the average credit card debt by a college student. Assume that the average credit card debt follows a normal distribution; $N(\mu, 10,000)$. They would like to narrow the value of the mean down to an interval of length \$10 with confidence level 95%. How large of a sample size must they use?

11. Independent random samples of the heights of adult males living in two countries yielded the following results: $n = 13$, $\overline{x} = 65.7$ inches, $s_x = 4$ inches and $m = 16$, $\overline{y} = 68.2$ inches, $s_y = 3$ inches. Find an approximate 98% confidence interval for the ratio of variances, σ_X^2/σ_Y^2. Does your confidence interval support or refute an assumption that the variances are equal? Explain.

12. **For the following circle the correct answer. Choices are capitalized, there is more than one choice to make per question.**

    (a) The number α denotes the probability of NOT REJECTING / REJECTING the null hypothesis (H_0) when the null hypothesis is TRUE / FALSE.

    (b) The number β denotes the probability of NOT REJECTING / REJECTING the null hypothesis (H_0) when the null hypothesis is TRUE / FALSE.

13. **Circle True or False:**

    (a) True False: A hypothesis test was designed to test $H_0 : \mu_X - \mu_Y = 0$ against $H_1 : \mu_X - \mu_Y > 0$. Suppose the test statistic led the researcher to reject H_0. If you construct a corresponding one-sided confidence interval for $\mu_X - \mu_Y > 0$ using the same confidence level, then you would expect the interval to contain 0.

(b) True False: If the critical region for a test is $z \leq -1.645$ and the observed value of z is -1.41 then H_0 should be rejected.

(c) True False: If a test of hypotheses is used where $\alpha = .01$ you are generally more likely to correctly accept the null hypothesis (e.g. accept H_0 when H_0 is true) than if $\alpha = .05$ (assuming all other aspects of the tests are equal).

Short Answer:

14. Suppose you wish to test $H_0 : \mu_x = \mu_0$ against $H_1 : \mu_x \neq \mu_0$ where σ^2 is known. Sketch a graph indicating the critical region for the appropriate test statistic with $\alpha = .1$. Label and give values for the cutoff for the critical region.

15. Suppose we believe a random variable X is binomial $b(n,p)$ and we wish to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{1}{4}$. If $n = 10$ and our critical region is $C = \{x : x \leq 2\}$, then find β, the probability of a type *II* error. $\beta =$ ______

Answers:

1. (a) reject H_0 ($z = 1.87$)
   (b) 0.0307
   (c) $(75.08, \infty)$ 75 is NOT in this interval.
   (d) They all consistently reject the notion that $\mu_X = 75$ in favor of the hypothesis that it is bigger than 75 with a confidence level of 0.05: the first by evaluating the test statistic to a value in the critical region; the second by getting a p-value which is less that 0.05; and the third by showing that 75 is too small to be in a one-sided 95% confidence interval for μ_X.

2. (a) $H_0 : \mu_X = \mu_Y$; $H_1 : \mu_X > \mu_Y$
   (b) $T = \dfrac{\overline{X} - \overline{Y}}{\sqrt{\frac{5(4.87)+10(5.70)}{15}\left(\frac{1}{6} + \frac{1}{11}\right)}}$ The critical region is $t \geq 1.753$
   (c) $t = 0.059$, do not reject H_0

3. Do not reject H_0 ($z = -1.32$ which is greater than $-z_{0.05} = -1.645$ so z is NOT in the critical region.)

4. (a) 51.90
   (b) 58.10
   (c) approximately 0.015

5. approx. 92.82%

6. 360

7. (a) $[0.3522, 0.3878]$
   (b) 1577

8. $[9.5, 22.3]$

9. $[0.2064, 0.2536]$ conclude that African women had a higher proportion of women with nutritional anemia

10. 1537

11. $[0.48, 7.12]$ Since 1 is in the interval, the interval supports the assumption of equal variances

12. (a) REJECTING/TRUE
    (b) NOT REJECTING/FALSE

13. (a) FALSE
    (b) FALSE
    (c) TRUE

14. I don't feel like drawing the picture here. I'm sure yours is lovely.

15. 0.4744