Math 366 Exam 1 Review Problems

• Exam 1 will cover §4.1 – §4.2 and §5.3 – §5.7. You may use **one side** of a full page of notes and all tables in the back of your book. You may also refer to the formulas in the front cover of your book.

1. At the Super Mall, it is known that 20% of customers use cash, 65% use Visa, and 15% use Mastercard. Select 3 customers at random and let X be the number who pay in cash and Y the number who pay with a Visa.
   (a) Find the joint p.m.f. of X and Y.
   (b) Find $P(X = 1, Y = 2)$
   (c) Find $Cov(X, Y)$
   (d) Find the correlation coefficient ρ.
   (e) Find the equation of the least squares regression line.

2. Let X and Y have joint p.d.f. given by

$$f(x, y) = 8xy, \qquad 0 \le x \le y \le 1$$

   (a) Find the marginal p.d.f. of Y.
   (b) Find μ_Y using the joint p.d.f..
   (c) Find μ_Y using the marginal p.d.f. of Y.
   (d) Find $P(X < 1/2)$.
   (e) Find $P(Y > 1/2)$.
   (f) Find $P(X \le 1/4; Y \ge 3/4)$.
   (g) Are X and Y independent? Explain.

3. Let X denote the number of hours per week that a college student spends watching TV. Suppose it is known that X is $N(11, 9)$.
   (a) Find the probability that a randomly chosen college student watches 5 or fewer hours of TV per week.
   (b) Find the probability that in a survey of 14 random college students the sample average is greater than or equal to 13.
   (c) Find the probability that in a survey of 14 random college students that the sum of their hours per week exceeds 182.

4. Let $\overline{X}$ be the mean of a random sample of size $n = 20$ from a Gamma distribution with $\alpha = 3$ and $\theta = 4$. Find $P(10.5 \le \overline{X} \le 12.7)$.

5. Suppose Y has the binomial distribution $b(100, .7)$. Use the normal distribution to estimate $P(80 \le Y \le 90)$.

6. Suppose that $X_1, X_2, .., X_n$ are independednt chi-square random variables with $r_1, r_2, .., r_n$ degrees of freedom respectively. Use the moment-generating function technique to prove that $Y = X_1 + X_2 + \cdots + X_n$ is $\chi^2(r_1 + r_2 + \cdots + r_n)$.

7. Let X and Y equal the respective number of hours a randomly selected woman or man spends watching t.v. per week. Suppose that it is known that X is $N(12, 100)$ and Y is $N(10, 225)$. If a man and woman are selected at random, find the probability that the woman watches more t.v. than the man.

8. Let X equal the weight of the soap in a "6-pound" box. Assume that the distribution of X is $N(6.05, 0.0004)$.
   (a) Find $P(X < 6.0171)$.
   (b) If nine boxes of soap are selected at random from the production line, find the probability that at most two boxes weigh less than 6.0171 each. Hint: Let Y equal the number of boxes that weigh less than 6.0171.
   (c) Let $\overline{X}$ be the sample mean of the nine boxes. Find $P(\overline{X} \leq 6.035)$.

9. Sam's car has a serious problem. After about every 20 miles, his right front tire seems to blow out and he has to put a new tire on. After detailed analysis, Sam determines that his right front tire has a lifetime X (time in hours it can be used before it blows and must be changed) that has a $N(21.23, 25)$ distribution. Sam is planning a 500 mile trip. How many tires should he bring so that with probability greater than 0.90 he has enough tires to complete his trip (assume no other tires besides his right front blow out)?

10. The time in minutes of a visit to a cardiovascular disease specialist by a patient is modeled by a gamma p.d.f. with $\alpha = 1.5$ and $\theta = 10$. Suppose that you are such a patient and you have four patients ahead of you. Assuming independence, what integral gives the probability that you will wait more than 90 minutes?

<u>Answers</u>

1. (a) $f(x, y) = \frac{3!}{x!y!(3-x-y)!}(0.2)^x(0.65)^y(0.15)^{3-x-y}$
   (b) 0.2535
   (c) -0.39
   (d) -0.681
   (e) $y = 2.44 - 0.812x$

2. (a) $f_1(y) = 4y^3$
   (b) $\mu_Y = 4/5$
   (c) $\mu_Y = 4/5$
   (d) $P(X < 1/2) = 7/16 = 0.4375$
   (e) $P(Y > 1/2) = 15/16 = 0.9375$
   (f) $P(x \leq 1/4; Y \geq 3/4) = 7/128 = 0.0547$
   (g) Dependent (non-rectangular support)

3. (a) 0.0228
   (b) 0.0064
   (c) 0.0064

4. 0.5076

5. 0.0192

6. See book page 244

7. 0.5438

8. (a) 0.05
   (b) 0.9916
   (c) 0.0122

9. 26 tires

10. $P(Y > 90) = \displaystyle\int_{90}^{\infty} \frac{1}{\Gamma(6)10^6} y^{6-1} e^{-y/10} dy$