Math 366: Exam 2 Review Sheet

Disclaimer: This is not meant to be an exhaustive set of examples. The exam will cover §6.3-6.6 and §7.1-7.5. I don’t have many problems from Ch. 6 in here.

1. A new method of teaching reading comprehension using a special computer game was developed and tested on 4-th graders. Let $X$ denote the score on a standardized exam of students taught using the computer game. Assume that $X$ is $N(\mu, 14)$ and test the hypothesis that $H_0 : \mu_x = 75$ against the alternative hypothesis $H_1 : \mu_x > 75$ with $\alpha = .05$.

(a) If in a sample of $n = 100$ 4-th graders, $\bar{x} = 75.7$, what is your conclusion?
(b) What is the p-value of the test?
(c) Find a one-sided 95% confidence interval giving a lower bound on $\mu_X$. Is 75 in this interval?
(d) Explain why your answers in a)-c) are consistent.

2. Let $X, Y$ denote the amount of bacteria on the bathroom counters and kitchen counters respectively of college dorms measured in number of bacteria colonies per square foot. In a study of 6 bathrooms and 11 kitchens from various college dorms, the following data was gathered:

$X$ (bathroom): 10.8, 7.2, 6.1, 12.0, 9.8, 9.1 ($\bar{x} = 9.16; s_x^2 = 4.87$)
$Y$ (kitchen): 12.1, 7.3, 4.9, 7.6, 11.0, 9.6, 12.7, 8.2, 8.5, 10.9, 7.2 ($\bar{y} = 9.09; s_y^2 = 5.70$)

(a) Suppose that $X$ is $N(\mu_x, \sigma^2_x)$ and $Y$ is $N(\mu_Y, \sigma^2_Y)$ and assume equal variances. State a null and alternative hypothesis comparing $\mu_X$ and $\mu_Y$, the mean number of bacteria colonies on the bathroom and kitchen counters.
(b) Define a test statistic and critical region using $\alpha = .05$.
(c) Evaluate the test statistic and state your conclusion.
(d) Test if the assumption of equal variances is valid using an $\alpha = .05$ significance level.

3. Let $p_m$ and $p_f$ denote the percentage of male and females respectively who drink coffee in the morning. Test

$H_0 : p_m = p_f$, against $H_1 : p_m < p_f$

if in a survey of 150 males, 83 drank coffee in the morning and in a survey of 125 females, 79 drank coffee in the morning. Use $\alpha = .05$.

4. The Fog Index is a measure of reading difficulty based on the average number of words per sentence and the percentage of words with three or more syllables. High values of teh Fog Index are associated with difficult reading levels. Independent random samples of four advertisements were taken from three different magazines and Fog Indexes were recorded. Hest the null hypothesis of no difference between mean Fog Index levels for advertisements in the three magazines using a significance level of $\alpha = 0.05$.

<table>
<thead>
<tr>
<th>Magazine</th>
<th>Scientific American</th>
<th>Fortune</th>
<th>New Yorker</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fog Index</td>
<td>15.75</td>
<td>11.55</td>
<td>11.16</td>
</tr>
<tr>
<td></td>
<td>12.63</td>
<td>11.46</td>
<td>10.77</td>
</tr>
<tr>
<td></td>
<td>9.27</td>
<td>8.28</td>
<td>8.15</td>
</tr>
</tbody>
</table>
5. Suppose, $X$ is $N(\mu_x, 100)$ We wish to test the hypothesis:

\[ H_0 : \mu_x = 50 \]
\[ H_1 : \mu_x > 50 \]

We take a sample of $n = 75$ and compute $\bar{X}$ to estimate $\mu_X$. Define the critical region as $C = \{ \bar{X} | \bar{X} \geq k \}$.

(a) What value should $k$ be if you want a significance level of $\alpha = 0.05$?
(b) What value should $k$ be if you want the probability of a type II error to be $\beta = 0.05$ when $\mu_X = 60$.
(c) What is the $p$-value of the test when $\bar{X} = 52.5$?

6. A total of 1154 girls attending a public high school were given a questionnaire that measured how much each had exhibited delinquent behavior. The following is a cross-classification of the delinquents and the nondelinquents according to their birth order. At the $\alpha = 0.05$ significance level test whether the birth order and delinquency are related.

<table>
<thead>
<tr>
<th>Birth Order</th>
<th>Delinquent</th>
<th>Not Delinquent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oldest</td>
<td>24</td>
<td>450</td>
</tr>
<tr>
<td>In Between</td>
<td>29</td>
<td>312</td>
</tr>
<tr>
<td>Youngest</td>
<td>35</td>
<td>211</td>
</tr>
<tr>
<td>Only Child</td>
<td>23</td>
<td>70</td>
</tr>
</tbody>
</table>

7. For the following circle the correct answer. Choices are capitalized, there is more than one choice to make per question.

(a) The number $\alpha$ denotes the probability of ACCEPTING / REJECTING the null hypothesis ($H_0$) when the null hypothesis is TRUE / FALSE.
(b) The number $\beta$ denotes the probability of ACCEPTING / REJECTING the null hypothesis ($H_0$) when the null hypothesis is TRUE / FALSE.

8. Circle True or False:

(a) True False: If a test is defined with $\alpha = 0.05$ and the $p$-value of the test statistic is $p$-value $= 0.045$, then $H_0$ should be rejected.

(b) True False: If the critical region for a test is $z \leq -1.645$ and the observed value of $z$ is $-1.41$ then $H_0$ should be rejected.

(c) True False: If a test of hypotheses is used where $\alpha = 0.01$ you are generally more likely to correctly accept the null hypothesis (e.g. accept $H_0$ when $H_0$ is true) than if $\alpha = 0.05$ (assuming all other aspects of the tests are equal).

Short Answer:

9. Suppose you wish to test $H_0 : \mu_x = \mu_0$ against $H_1 : \mu_x \neq \mu_0$ where $\sigma^2$ is known. Sketch a graph indicating the critical region for the appropriate test statistic with $\alpha = .1$. Label and give values for the cutoff for the critical region.

10. Suppose we believe a random variable $X$ is binomial $b(n, p)$ and we wish to test $H_0 : p = \frac{1}{4}$ against $H_1 : p = \frac{1}{4}$. If $n = 10$ and our critical region is $C = \{ x : x \leq 2 \}$, then find $\beta$, the probability of a type II error. $\beta =$

11. Explain the meaning of $p$-value.
Answers:

1. (a) reject \( H_0 \) \((z = 1.87)\)
   (b) 0.0307
   (c) \((75.08, \infty)\) 75 is NOT in this interval.
   (d) They all consistently reject the notion that \( \mu_X = 75 \) in favor of the hypothesis that it is bigger than 75 with a confidence level of 0.05: the first by evaluating the test statistic to a value in the critical region; the second by getting a p-value which is less than 0.05; and the third by showing that 75 is too small to be in a one-sided 95% confidence interval for \( \mu_X \).

2. (a) \( H_0 : \mu_X = \mu_Y ; H_1 : \mu_X > \mu_Y \)
   (b) The t statistic with the complicated formula which I'm too lazy to type right now. The critical region is \( t \geq 1.753 \)
   (c) \( t = 0.059 \), do not reject \( H_0 \)
   (d) Yes, the equal variance assumption is valid (remember - this is a two sided test!)

3. Do not reject \( H_0 \) \((z = -1.32 \) which is greater than \(-z_{0.05} = -1.645 \) so \( z \) is NOT in the critical region.)

4. Reject \( H_0 \) (Note the test statistic is \( F = 6.0 \))

5. (a) 51.90
   (b) 58.10
   (c) approximately 0.015

6. Do not accept \( H_0 \)

7. (a) REJECTING/TRUE
   (b) ACCEPTING/FALSE

8. (a) TRUE
   (b) FALSE
   (c) TRUE

9. I don’t feel like drawing the picture here. I’m sure yours is lovely.

10. 0.4744

11. We discussed this in class. Explain it in your own words.