5.4 Central Limit Theorem

$$W = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0,1)$$

Problem 5.4-7

X: $\mu = 54.030$

$\sigma = 5.8$

$n = 47$

find $P(52.761 \le \bar{X} \le 54.453)$

$$= P\left(\frac{52.761 - 54.030}{5.8/\sqrt{47}} \le W \le \frac{54.453 - 54.030}{5.8/\sqrt{47}}\right)$$

$= P(W \le .5) - P(W \ge 1.5)$

$= .6915 - .0668$ (from Normal Table)

$= \boxed{.6247}$

5.5 Approximations for discrete distributions

$$W = \frac{Y - np}{\sqrt{np(1-p)}} \sim N(0,1)$$

5.5-1 (b)

let $Y \sim b(25, 1/2)$

Approximate $P(12 \le Y < 15)$

$$= P\left(\frac{11.5 - 25(1/2)}{\sqrt{25(1/2)(1/2)}} \le W \le \frac{14.5 - 25(1/2)}{\sqrt{25(1/2)(1/2)}}\right)$$

$= P(W \le .8) - P(W \ge .4)$

From normal table...

$= .7881 - .3446$

$\boxed{= .4435}$

part 2: Poisson distribution (w/ mean λ)

$$W = \frac{Y - \lambda}{\sqrt{\lambda}} \sim N(0,1)$$

6.1

Basic Idea: Given set of <u>ordered</u> data, n items.

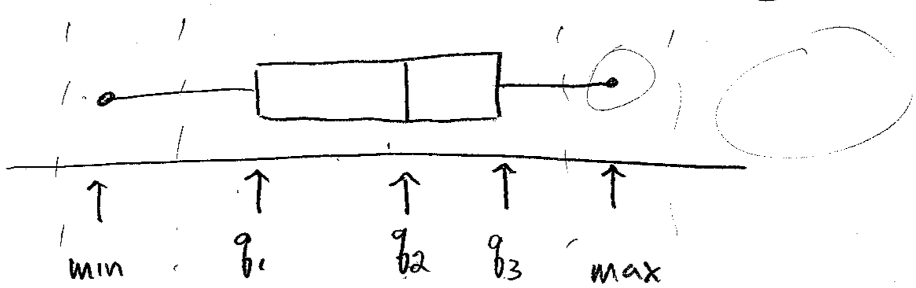
5 number summary

		min
Number that a certain percent of data falls below.	.25	q_1
	.50	q_2
	.75	q_3
		max

$p(n+1) = I + d$

Then use $(1-d)Y_i + d \cdot Y_{i+1}$

Box + Whiskers: Using 5 number summary



Fences: $IQR = q_3 - q_2$

suspected outliers → Inner: [illegible] are 1.5 IQR from both sides of our box.

$q_1 - 1.5IQR$, $q_3 + 1.5IQR$

outliers → Outer: $3 \cdot IQR$ from both sides

$q_1 - 3IQR$ $\quad q_3 + 3IQR$

6.2

Maximum Likelihood Estimator

1. Form the function $L(\theta_1, \theta_2, \ldots, \theta_n)$
2. Simplify
3. Take $\ln(L(\theta))$
4. derive θ

ex: p.d.f. is $\theta x^{\theta-1}$ $0 \leq x \leq 1$

$$L(\theta) = \prod_{i=1}^{n} \theta x_i^{\theta-1}$$

$$= \theta^n x_i^{\theta-1}$$

$$\ln(L(\theta)) = \ln(\theta^n) + \ln\left(\prod_{i=1}^{n} x_i^{\theta-1}\right)$$

$$= n \ln \theta + (\theta - 1) \ln \prod_{i=1}^{n} x_i$$

$$= n \ln \theta + \theta \sum_{i=1}^{n} \ln x_i - \sum_{i=1}^{n} \ln x_i$$

$$\frac{d}{d\theta}\left(\ln(L(\theta))\right) = \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i$$

$$0 = \frac{n}{\theta} + \sum_{i=1}^{n} \ln x_i$$

$$\theta = \frac{-n}{\sum \ln x_i}$$

Method of Moments

is a method of estimation such as mean, variance etc. by equating sample moments with unobservable moments ; then solving these equations for quantities to be estimated

$$\sigma^2 + \mu^2 = E[x^2]$$

$$\sigma^2 + \mu^2 = \frac{1}{n} \sum x_i^2$$

biased vs. unbiased estimators - suppose we find that $\hat{\theta}$ is an estimator for θ. Then $\hat{\theta}$ is unbiased if its expected value is actually θ. That is $E[\hat{\theta}] = \theta$

16) $\mu = \bar{X} = 6.74$ $\alpha\theta = \bar{X}$ $\alpha\theta^2 + \alpha^2\theta^2 = \frac{1}{n}\sum x_i^2$

$\sigma^2 + \mu^2 = \frac{1}{n}\sum x_i^2$ $\sigma^2 = \alpha\theta^2$ $\alpha = \frac{6.74}{\theta}$

$\sigma^2 + \mu^2 = \frac{1}{25} \cdot 1146.77$

$$\left(\frac{6.74}{\theta}\right)\theta^2 + \left(\frac{6.74}{\theta}\right)^2\theta^2 = 45.8708$$

$\theta = .0657$

$\alpha = 102.499$

$\boxed{\text{6.4 Confidence Intervals For Means}}$ 6.4 + 6.5

Confidence Interval

$$P\left[\bar{X} - z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{X} + z_{\alpha/2}\left(\frac{\sigma}{\sqrt{n}}\right)\right] = \underbrace{(1-\alpha)100\%}_{\text{confidence coefficient}}$$

One-Sided Confidence Interval

$$P\left[\bar{X} - z_{\alpha}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu\right] = (1-\alpha)100\%$$

★ If σ^2 is unknown, then use t-student test statistic.

Important Concept: A shorter confidence interval indicates we have more reliance in $\bar{X}$ as an estimate of μ.

$\boxed{\text{6.5 Confidence Intervals for Difference of Two Means.}}$

$$P\left(-z_{\alpha/2} \le \frac{(\bar{X}-\bar{Y}) - (\mu_x - \mu_y)}{\sqrt{(\sigma_x^2/n) + (\sigma_y^2/m)}} \le z_{\alpha/2}\right) = (1-\alpha)100\%$$

or $\bar{x} - \bar{y} \pm z_{\alpha/2}\sqrt{\frac{\sigma_x^2}{n} + \frac{\sigma_y^2}{m}}$

unknown variances + large samples use:

$$\bar{x} - \bar{y} \pm z_{\alpha/2}\sqrt{\frac{s_x^2}{n} + \frac{s_y^2}{m}}$$

unknown variances + assume $\sigma_x^2 = \sigma_y^2$ use:

$$\bar{x} - \bar{y} \pm t_{\alpha/2}(n+m-2)\, s_p\sqrt{\frac{1}{n} + \frac{1}{m}}, \quad s_p = \sqrt{\frac{(n-1)s_x^2 + (m-1)s_y^2}{n+m-2}}$$

X + Y are dependent use:

$(\mu_D = \mu_x - \mu_y)$ $\bar{d} \pm t_{\alpha/2}(n-1)\frac{s_d}{\sqrt{n}}$

6.7 Key Ideas / Theorems

Confidence interval for proportions when n is large

$$\frac{y}{n} \pm z_{\frac{\alpha}{2}} \sqrt{\frac{(y/n)(1-y/n)}{n}}$$

upper bound $= \left[0, \frac{y}{n} + z_{\alpha}\sqrt{\frac{y/n\,(1-y/n)}{n}}\right]$

lower bound $= \left[\frac{y}{n} - z_{\alpha}\sqrt{\frac{y/n\,(1-y/n)}{n}}, 1\right]$

multiple experiments $= \frac{y_1}{n_1} - \frac{y_2}{n_2} \pm \sqrt{\frac{(y_1/n_1)(1-y_1/n_1)}{n_1} + \frac{(y_2/n_2)(1-y_2/n_2)}{n_2}}$

6.7 Example #14 (a, b, c, d)

(a) $\hat{p}_1 = \frac{y_1}{n_1} = \frac{28}{194} \approx .1443$

(b) $C.I. = .1443 \pm Z_{.25}\sqrt{\frac{(.1443)(1-.1443)}{194}} = .1443 \pm 1.96(.0252)$

$= (-.0949, .1937)$

(c) $\frac{28}{194} - \frac{11}{162} = .0764$

(d) $.0764 \pm Z_{.25}\sqrt{\frac{(.1443)(1-.1443)}{194} + \frac{(.0679)(1-.0679)}{162}}$

$= .0764 \pm 1.96(.0320)$
$= (.0137, .1391)$

6.8 Key Ideas / Theorems

ε = maximum error of the estimate.

$\varepsilon = \frac{Z_{\frac{\alpha}{2}}\sigma}{\sqrt{n}}$, where $\phi(Z_{\frac{\alpha}{2}}) = 1 - \frac{\alpha}{2}$

$n = \frac{Z_{\frac{\alpha}{2}}\sigma^2}{\varepsilon^2}$, where the variance is known and it is a normal dist

$n = \frac{Z^2_{\frac{\alpha}{2}}}{4\varepsilon^2}$, where it is a binomial dist and p is unknown

$n = \frac{Z^2_{\frac{\alpha}{2}} p^*(1-p^*)}{\varepsilon^2}$, where it is a binomial dist and p^* is close to the probability

6.8 Example # 9

$$n = \frac{Z^2_{\frac{.01}{2}} \left(\frac{1}{6}\right)\left(\frac{5}{6}\right)}{(.02)^2} = \frac{.9216}{(.02)^2} = 2304.0\overline{888}$$

$$\lceil 2304.0\overline{888} \rceil = 2305$$

§8.1 → Test of statistical Hypothesis

Type I error: Rejecting H_0 & Accepting H_1 when H_0 is true

Type II error: Failing to reject H_0 when H_1 is true, that is, when H_0 is false.

- Finding the critical region
- Test of Hypothesis for one proportion
- Finding p-values
- Test of Hypotheses for two-proportions

8.1 #7

H_0: $p = 0.07$ $\quad H_1$: $p > 0.07$

$y = 23 \quad n = 209$

(a) $\alpha = 0.05$? $\Rightarrow$ $z = \frac{y/n - p_0}{\sqrt{p_0(1-p_0)/n}} \geq z_\alpha$

$$\frac{\frac{23}{209} - 0.07}{\sqrt{\frac{0.07(0.93)}{209}}} \geq z_\alpha$$

$z_{0.05} = 1.645$

$2.27 \geq 1.645$

Accept H_1

(b) $\alpha = 0.01$? $\quad z_{0.01} = 2.326$

$2.27 \not\geq 2.326$ Accept H_0

(c) p-value $\quad P(x \geq 2.27) = 1 - P(x \leq 2.27) = 1 - 0.9884$

p-value $= 0.0116$

8.2 Key ideas: H_0 vs H_1 for μ

Using Normal Distribution: $\frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \geq z_\alpha \quad \sigma^2$ known

Using t Distribution: $\frac{\bar{x} - \mu_0}{s/\sqrt{n}} \geq t_\alpha(n-1) \quad \sigma^2$ unknown

H_0 vs H_1 for σ^2

Using χ^2: $\frac{(n-1)s^2}{\sigma_0^2} \geq \chi^2_\alpha(n-1)$

8.3.3

$n=9 \quad \bar{x}=21.03 \quad S_x=.60 \quad min=20.2 \quad q_1=20.5 \quad med=21 \quad q_3=2$
$max=21.9$

$m=13 \quad \bar{y}=20.89 \quad S_y=1 \quad min=18.9 \quad q_1=20.3 \quad med=20.8 \quad q_3=$
$max=23$

$H_0 \; \frac{\sigma_x^2}{\sigma_y^2}=1 \qquad \frac{S_x^2}{S_y^2} = \frac{.36}{1} = .36 \qquad \frac{S_y^2}{S_x^2} = 2.77$

at $\alpha = .05 \qquad F_{.025}(12,8) = 4.20 > 2.77$

$F_{.025}(8,12) = 3.51 > .36$

We should then accept the null hypothesis $\frac{\sigma_x^2}{\sigma_y^2}=1$

Test if $H_0 \; \mu_x = \mu_y$ at $\alpha = .05$

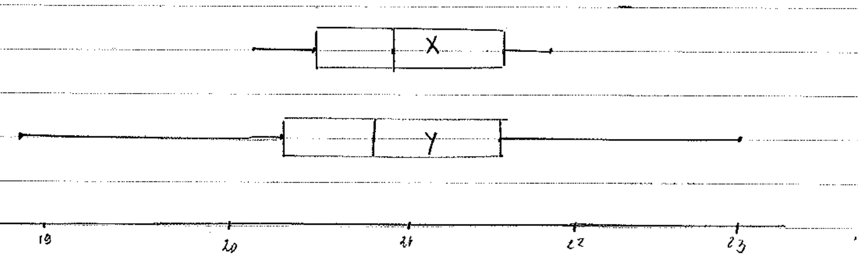
Since $(n+m-2) < 30$ we should use the formula

$$\frac{\bar{x}-\bar{y}}{S_p\sqrt{\frac{1}{n}+\frac{1}{m}}}$$

$$t = \frac{|21.03-20.89|}{\sqrt{\frac{8\times.36+12\times1}{20}}\times\sqrt{\frac{1}{9}+\frac{1}{12}}} = .3681$$

We have $t_{.025}(20) = 2.086$

$.368 < 2.086$ We should accept H_0 thus $\mu_x = \mu_y$



§ 8.5 Chi-Square Goodness of Fit Tests

A Chi-Square Test can be used to test hypotheses about distributions of data.

Theorem 5.2-2: If $Z \sim N(0,1)$, then $Z^2 \sim \chi^2(1)$

Theorem 5.2-3: If Z_i are $N(0,1)$, then $Z_1^2 + Z_2^2 + \dots + Z_n^2 \sim \chi^2(n)$

$$Q_{k-1} = \sum_{i=1}^{k} \frac{(y_i - np_i)^2}{np_i} \sim \chi^2(k-1) \quad \text{where:}$$

y_j is the number of successes for the j^{th} outcome

p_j is the probability of y_j successes

n is the total number of trials

k is the number of possible outcomes

Example) Problem 6, pg. 530

In a biology laboratory students test the Mendelian theory of inheritance using corn. The Mendelian theory of inheritance claims that frequencies of the four categories smooth and yellow, wrinkled and yellow, smooth and purple, and wrinkled and purple will occur in the ratio 9:3:3:1. If a student counted 124, 30, 43, and 11, respectively, for these four categories, would these data support the Mendelian Theory? Let $\alpha = 0.05$.

Outcome	Observed ~~Frequency~~ (y_i)	Expected Probability (p_i)	Expected Outcome (np_i)
smooth and yellow	124	$9/16 = .5625$	117
wrinkled and yellow	30	$3/16 = .1875$	39
smooth and purple	43	$3/16 = .1875$	39
wrinkled and purple	11	$1/16 = .0625$	13

$$Q_3 = \frac{(124-117)^2}{117} + \frac{(30-39)^2}{39} + \frac{(43-39)^2}{39} + \frac{(11-13)^2}{13} = 3.214$$

$\chi^2_{.05}(3) = 7.815$

$3.214 < 7.815$

Accept H_0

8.6 Contingency Tables

- Can be used to see if two or more multinomial distributions are equal (homogeneous).
- Can be used to test the independence of experimental attributes.

Multinomial distributions

Suppose we have $B_1, B_2, \ldots, B_h$ multinomial distributions with $A_1, A_2, \ldots, A_k$ outcomes, such that $A_1, A_2, \ldots, A_k$ are mutually exclusive and exhaustive.

	A_1	A_2	$\ldots$	A_k	Total
B_1	y_{11}	y_{12}	$\ldots$	y_{1k}	n_1
B_2	y_{21}	y_{22}	$\ldots$	y_{2k}	n_2
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$	$\vdots$
B_h	y_{h1}	y_{h2}	$\ldots$	y_{hk}	n_j

Let y_{ij} denote the number of observations of A_i in distribution B_j and p_{ij} denote the probability of outcome A_i, $P(A_i)$. Supposing that each experiment $B_1, B_2, \ldots, B_h$ is repeated $n_1, n_2, \ldots, n_h$ times respectively, then the test statistic Q is:

$$Q = \sum_{j=1}^{h} \sum_{i=1}^{k} \frac{(y_{ij} - n_j p_{ij})^2}{n_j p_{ij}} \sim \chi^2((k-1)(h-1))$$

$\left(p_{ij} = \dfrac{y_{i1} + y_{i2} + y_{i3} + \ldots + y_{ih}}{n_1 + n_2 + n_3 + \ldots + n_h} \right.$, i.e. the sum of a column over total sample size$\left.\right)$

We test Q against $\chi^2_\alpha((k-1)(h-1))$.

Example: Suppose that there are 270 musicians auditioning for three different musical groups-- the low, middle, and high-skilled. They are broken into three equal groups A, B, and C to perform group auditions from which they will be selected and placed into one of the three musical ensembles. Given the following data, does the group that a musician was placed in have an effect on which ensemble they made? Use $\alpha = .05$

	Low	Middle	High
A	27	32	31
B	40	31	19
C	35	32	23

$P(\text{Low}) = \frac{27+40+35}{90+90+90} = \frac{102}{270} = .378$

$P(\text{Middle}) = \frac{32+31+32}{90+90+90} = \frac{95}{270} = .352$

$P(\text{High}) = \frac{31+19+23}{90+90+90} = \frac{73}{270} = .270$

$H_0: P(\text{High}) = P(\text{Middle}) = P(\text{High})$

$$Q = \frac{(27-90(.378))^2}{90(.378)} + \frac{(32-90(.352))^2}{90(.352)} + \frac{(31-90(.270))^2}{90(.270)}$$

$$+ \frac{(40-90(.378))^2}{90(.378)} + \frac{(31-90(.352))^2}{90(.352)} + \frac{(19-90(.270))^2}{90(.270)}$$

$$+ \frac{(35-90(.378))^2}{90(.378)} + \frac{(32-90(.352))^2}{90(.352)} + \frac{(23-90(.270))^2}{90(.270)} = 5.621$$

$$\chi^2_{.05}((k-1)(h-1)) = \chi^2_{.05}((3-1)(3-1)) = \chi^2_{.05}(2 \cdot 2) = \chi^2_{.05}(4) = 9.488$$

Since $5.621 < 9.488$, we accept H_0 and the distributions are equal between the three groups.

$\boxed{8.7}$

Source	SS	DOF	MS	F Ratio
Treatment	$SS(T)$	$m-1$	$SS(T)/_{m-1}$	$MS(T)/MS(E)$
Error	$SS(E)$	$n-m$	$SS(E)/_{n-m}$	
Total	$SS(TO)$	$n-1$		

$$SS(T) = \sum_{i=1}^{m} n_i(\bar{X}_{i.} - \bar{X}_{..})^2$$

$m = \#$ of rows

$n = \#$ of observations

$$SS(E) = \sum_{i=1}^{m} \sum_{j=1}^{n_i} (X_{ij} - \bar{X}_{i.})^2$$

$H_0: \mu_1 = \mu_2 = \ldots = \mu_m$

H_1: Not all μ_i are equal $1 \le i \le m$

$$SS(TO) = SS(E) + SS(T)$$

C.R.: $F_\alpha(m-1, n-m) \le MS(T)/MS(E)$

Examples

$\boxed{8.7-4}$ For the following set of data, show that the computed $SS[E]/(n-m) = 1$ & $SS[T]/(m-1) = 75$. Test F with $\alpha = .05$.

				Mean	
X_1:	4	5	6	5	
X_2:	9	10	11	10	10
X_3:	14	15	16	15	

Sol. $SS(T) = 3(5-10)^2 + 3(10-10)^2 + 3(15-10)^2 = \boxed{150}$

$SS(E) = (4-5)^2 + (5-5)^2 + (6-5)^2 + (9-10)^2 + (10-10)^2 + (11-10)^2 + (14-15)^2 + (15-15)^2 + (16-15)^2 = \boxed{6}$

Source	SS	DoF	MS	F
Treatment	150	2	75	75
Error	6	6	1	

$F_{.05}(2,6) = 5.14$ Since $5.14 < 75$, reject H_0.

§8.8 2-factor analysis of variance (one observation per cell)

Source	(SS)	Dof	Ms	F
factor A	SS(A)	$a-1$	$MS(A) = \frac{SS(A)}{a-1}$	$\frac{MS(A)}{MS(E)}$
Factor B	SS(B)	$b-1$	$MS(B) = \frac{SS(B)}{b-1}$	$MS(B)/MS(E)$
Error	SS(E)	$(a-1)(b-1)$	$MS(E) = \frac{SS(E)}{(a-1)(b-1)}$	
Total	SS(TO)	$ab-1$		

a = rows
b = columns

CR H_A $F_\alpha(\text{dof } A, \text{dof } E)$

CR H_B $F_\alpha(\text{dof } B, \text{dof } E)$

$$SS(A) = b\sum_{i=1}^{a}(\bar{X}_{i\cdot} - \bar{X}_{\cdot\cdot})^2$$

$$SS(B) = a\sum_{j=1}^{b}(\bar{X}_{\cdot j} - \bar{X}_{\cdot\cdot})^2$$

$$SS(E) = \sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{X}_{ij} - \bar{X}_{i\cdot} - \bar{X}_{\cdot j} + \bar{X}_{\cdot\cdot})^2$$

$$SS(TO) = SS(A) + SS(B) + SS(E)$$

	Gasoline				
Car	1	2	3	4	$\bar{X}_{i\cdot}$
1	16	18	21	21	19
2	14	15	18	17	16
3	15	15	18	16	16
$\bar{X}_{\cdot j}$	15	16	19	18	17

$SS(A) = 4[(19-17)^2 + (16-17)^2 + (16-17)^2] = 24$
$SS(B) = 3[(15-17)^2 + (16-17)^2 + (19-17)^2 + (18-17)] = 30$
$SS(E) = (16-19-15+17)^2 + (14-16-15+17)^2 + \ldots + (16-16-18+17)^2 = 4$
$SS(T) = SS(A) + SS(B) + SS(E)$

$F_A = \frac{24}{2} / \frac{4}{6} = 18$ Compare to: $F_\alpha(\text{dof } A, \text{dof } E)$

$F_B = \frac{30}{3} / \frac{4}{6} = 15$ Compare to: $F_\alpha(\text{dof } B, \text{dof } E)$

$\alpha = .05$
$18 > 5.14$ reject H_A (cars give different performances)
$15 > 4.76$ reject H_B (gas gives different performances)

Two way ANOVA Table, c Observations Per cell

Source	(SS)	Dof	(MS)	F
Factor A (row)	SS(A)	$a-1$	$MS(A)=\frac{SS(A)}{a-1}$	$\frac{MS(A)}{MS(E)}$
Factor B (column)	SS(B)	$b-1$	$MS(B)=\frac{SS(B)}{b-1}$	$\frac{MS(B)}{MS(E)}$
Factor AB (interaction)	SS(AB)	$(a-1)(b-1)$	$MS(AB)=\frac{SS(AB)}{(a-1)(b-1)}$	$\frac{MS(AB)}{MS(E)}$
Error	SS(E)	$ab(c-1)$	$MS(E)=\frac{SS(E)}{ab(c-1)}$	
Total	SS(TO)	$abc-1$		

$$\left.\begin{aligned} F_{AB} &\geq F_\alpha[(a-1)(b-1),\, ab(c-1)] \\ F_A &\geq F_\alpha[a-1,\, ab(c-1)] \\ F_B &\geq F_\alpha[b-1,\, ab(c-1)] \end{aligned}\right\} \text{critical regions}$$

$$SS(A) = bc\sum_{i=1}^{a}(\bar{X}_{i..}-\bar{X}_{...})^2$$

$$SS(B) = ac\sum_{j=1}^{b}(\bar{X}_{.j.}-\bar{X}_{...})^2$$

$$SS(AB) = c\sum_{i=1}^{a}\sum_{j=1}^{b}(\bar{X}_{ij.}-\bar{X}_{i..}-\bar{X}_{.j.}+\bar{X}_{...})^2$$

$$SS(E) = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{c}(X_{ijk}-\bar{X}_{ij.})^2$$

$$SS(TO) = SS(A)+SS(B)+SS(AB)+SS(E)$$

9.1 Key Ideas

$K = P(\text{Rejecting } H_0, \text{ when } H_0 \text{ is true})$

- K is called the Power function of the Test
- The value of K at p is called the "power of the test at p" $(K(p))$
- can use K to select sample size, n, and critical region, c.

Example Time!!!

9.1-7 X is # of lbs of butterfat produced by a cow blah blah blah... milking period... blah calf.

$X \sim N(\mu, 140^2)$

$H_0: \mu = 715$ $\quad\quad C = \{\bar{x} : \bar{x} \le c\}$

$H_1: \mu < 715$

Find n and c such that $\alpha = .05$ and $K(650) = .9$

Solution

$K(\mu) = P(\bar{X} \le c : \mu)$ ← power function

* $\alpha = .05$

$K(715) = P\left(Z \le \frac{\sqrt{n}(c-715)}{140}\right) = \Phi\left(\frac{\sqrt{n}(c-715)}{140}\right) = .05$

$-1.645 = \frac{\sqrt{n}(c-715)}{140}$

$-230.3\, n^{-1/2} + 715 = c \quad (1)$

* $K(650) = .90$

$K(650) = P\left(Z \le \frac{\sqrt{n}(c-650)}{140}\right) = \Phi\left(\frac{\sqrt{n}(c-650)}{140}\right) = .9$

$1.282 = \frac{\sqrt{n}(c-650)}{140}$

$179.48\, n^{-1/2} + 650 = c \quad (2)$

by (1) and (2)

$179.48\, n^{-1/2} + 650 = -230.3\, n^{-1/2} + 715$

$179.48 + 650\, n^{1/2} = -230.3 + 715\, n^{1/2}$

$409.78 = 65\, n^{1/2}$

$(409.78/65)^2 = (n^{1/2})^2$

$39.7 = n$

$\therefore n \approx 40$

by (2) $\quad 179.48\,(40)^{-1/2} + 650 = 678.38 = c$

$\therefore c = 678.38$