

Disclaimer: This is not meant to be an exhaustive set of examples. Study all sections we have covered so far in Chapter 8 (8.1-8.3, 8.5-8.7)

1. A new method of teaching reading comprehension using a special computer game was developed and tested on 4-th graders. Let X denote the score on a standardized exam of students taught using the computer game. Assume that X is $N(\mu, 14)$ and test the hypothesis that $H_0 : \mu_x = 75$ against the alternative hypothesis $H_1 : \mu_x > 75$ with $\alpha = .05$.

- (a) If in a sample of $n = 100$ 4-th graders, $\bar{x} = 75.7$, what is your conclusion?
- (b) What is the p-value of the test?
- (c) Find a one-sided 95% confidence interval giving a lower bound on μ_X . Is 75 in this interval?
- (d) Explain why your answers in a)-c) are consistent.

2. Let X, Y denote the amount of bacteria on the bathroom counters and kitchen counters respectively of college dorms measured in number of bacteria colonies per square foot. In a study of 6 bathrooms and 11 kitchens from various college dorms, the following data was gathered:

X (bathroom): 10.8, 7.2, 6.1, 12.0, 9.8, 9.1 ($\bar{x} = 9.16$; $s_x^2 = 4.87$)

Y (kitchen): 12.1, 7.3, 4.9, 7.6, 11.0, 9.6, 12.7, 8.2, 8.5, 10.9, 7.2 ($\bar{y} = 9.09$; $s_y^2 = 5.70$)

- (a) Suppose that X is $N(\mu_x, \sigma_x^2)$ and Y is $N(\mu_Y, \sigma_Y^2)$ and assume equal variances. State a null and alternative hypothesis comparing μ_X and μ_Y , the mean number of bacteria colonies on the bathroom and kitchen counters.
 - (b) Define a test statistic and critical region using $\alpha = .05$.
 - (c) Evaluate the test statistic and state your conclusion.
 - (d) Test if the assumption of equal variances is valid using an $\alpha = .05$ significance level.
3. The U-Can-Win Lottery boasts that the probability of a \$5 win is 1 in 4, the probability of a \$50 win is 1 in 10 and the probability of a \$500 win is 1 in 100. Use the Chi-Square goodness of fit test to determine if the lottery claims are accurate if 500 tickets were purchased and 150 were \$5 winners, 40 were \$50 winners, 3 were \$500 winners and 307 were losing tickets. Use $\alpha = .05$. Clearly state your conclusion.

4. Let p_m and p_f denote the percentage of male and females respectively who drink coffee in the morning. Test

$$H_0 : p_m = p_f, \text{ against } H_1 : p_m < p_f$$

if in a survey of 150 males, 83 drank coffee in the morning and in a survey of 125 females, 79 drank coffee in the morning. Use $\alpha = .05$.

5. The Fog Index is a measure of reading difficulty based on the average number of words per sentence and the percentage of words with three or more syllables. High values of the Fog Index are associated with difficult reading levels. Independent random samples of four advertisements were taken from three different magazines and Fog Indexes were recorded. Test the null hypothesis of no difference between mean Fog Index levels for advertisements in the three magazines using a significance level of $\alpha = 0.05$.

Scientific American	15.75	11.55	11.16	9.92
Fortune	12.63	11.46	10.77	9.93
New Yorker	9.27	8.28	8.15	6.37

6. Suppose, X is $N(\mu_x, 100)$ We wish to test the hypothesis:

$$H_0 : \mu_X = 50$$

$$H_1 : \mu_X > 50$$

We take a sample of $n = 75$ and compute \bar{X} to estimate μ_X . Define the critical region as $C = \{\bar{x} \mid \bar{x} \geq k\}$.

- What value should k be if you want a significance level of $\alpha = 0.05$?
 - What value should k be if you want the probability of a type II error to be $\beta = 0.05$ when $\mu_X = 60$.
 - What is the p -value of the test when $\bar{x} = 52.5$?
7. A total of 1154 girls attending a public high school were given a questionnaire that measured how much each had exhibited delinquent behavior. The following is a cross-classification of the delinquents and the nondelinquents according to their birth order. At the $\alpha = 0.05$ significance level test whether the birth order and delinquency are related.

	Delinquent	Not delinquent
Oldest	24	450
In Between	29	312
Youngest	35	211
Only Child	23	70

8. **For the following circle the correct answer. Choices are capitalized, there is more than one choice to make per question.**

- The number α denotes the probability of ACCEPTING / REJECTING the null hypothesis (H_0) when the null hypothesis is TRUE / FALSE.
- The number β denotes the probability of ACCEPTING / REJECTING the null hypothesis (H_0) when the null hypothesis is TRUE / FALSE.

9. **Circle True or False:**

- True False: If a test is defined with $\alpha = .05$ and the p -value of the test statistic is p -value = .045, then H_0 should be rejected.
- True False: If the critical region for a test is $z \leq -1.645$ and the observed value of z is -1.41 then H_0 should be rejected.
- True False: If a test of hypotheses is used where $\alpha = .01$ you are generally more likely to correctly accept the null hypothesis (e.g. accept H_0 when H_0 is true) than if $\alpha = .05$ (assuming all other aspects of the tests are equal).

Short Answer:

- Suppose you wish to test $H_0 : \mu_x = \mu_0$ against $H_1 : \mu_x \neq \mu_0$ where σ^2 is known. Sketch a graph indicating the critical region for the appropriate test statistic with $\alpha = .1$. Label and give values for the cutoff for the critical region.
- Suppose we believe a random variable X is binomial $b(n, p)$ and we wish to test $H_0 : p = \frac{1}{2}$ against $H_1 : p = \frac{1}{4}$. If $n = 10$ and our critical region is $C = \{x : x \leq 2\}$, then find β , the probability of a type II error. $\beta = \underline{\hspace{2cm}}$
- Explain the meaning of p -value.

More Practice Problems

These problems were taken directly from odd textbook problems. Next to the problem is the section of the book and problem number so you can look up the answer in your book. For even more practice, make a similar set of problems using the textbook and trade with a study partner.

- (§8.2#17) Each of 51 golfers hit three golf balls of brand X and three golf balls of brand Y in a random order. Let X_i and Y_i equal the averages of the distances traveled by the brand X and brand Y golf balls hit by the i th golfer, $i = 1, 2, \dots, 51$. Let $W_i = X_i - Y_i$, $i = 1, 2, \dots, 51$. Test $H_0 : \mu_W = 0$ against $H_1 : \mu_W > 0$, where μ_W is the mean of the differences. If $\bar{w} = 2.07$ and $s_w^2 = 84.63$, would H_0 be accepted or rejected at an $\alpha = 0.05$ significance level?
- (§8.1#19) Let p_m and p_f be the respective proportions of male and female white-crowned sparrows that return to their hatching site. Give the endpoints for a 95% confidence interval for $p_m - p_f$, given that 124 out of 894 males and 70 out of 700 females returned. Does this agree with the conclusion of a test of $H_0 : p_1 = p_2$ against $H_1 : p_1 \neq p_2$ with $\alpha = 0.05$?
- (§8.3#17) To measure air pollution in a home, let X and Y equal the amount of suspended particulate matter (in $\mu\text{g}/\text{m}^3$) measured during a 24-hour period in a home in which there is no smoker and a home in which there is a smoker, respectively. (Assume that the distributions of the independent random variables X and Y are $N(\mu_X, \sigma_X^2)$ and $N(\mu_Y, \sigma_Y^2)$, respectively.) We shall test the null hypothesis $H_0 : \sigma_X^2/\sigma_Y^2 = 1$ against the one-sided alternative hypothesis $H_1 : \sigma_X^2/\sigma_Y^2 > 1$. If a random sample of size $n = 9$ yielded $\bar{x} = 93$ and $s_x = 12.9$ while a random sample of size $m = 11$ yielded $\bar{y} = 132$ and $s_y = 7.1$, define a critical region and give your conclusion if $\alpha = 0.05$. Now test $H_0 : \mu_X = \mu_Y$ against $H_1 : \mu_X < \mu_Y$ if $\alpha = 0.05$.
- Section 8.5 Problems 5, 9
- Section 8.6 Problems 7,13
- Section 8.7 Problems 3,7,11

Answers:

- (a) reject H_0 ($z = 1.87$)
 (b) 0.0307
 (c) $(75.08, \infty)$ 75 is NOT in this interval.
 (d) They all consistently reject the notion that $\mu_X = 75$ in favor of the hypothesis that it is bigger than 75 with a confidence level of 0.05: the first by evaluating the test statistic to a value in the critical region; the second by getting a p-value which is less than 0.05; and the third by showing that 75 is too small to be in a one-sided 95% confidence interval for μ_X .
- (a) $H_0 : \mu_X = \mu_Y$; $H_1 : \mu_X > \mu_Y$
 (b) The t statistic with the complicated formula which I'm too lazy to type right now. The critical region is $t \geq 1.753$
 (c) $t = 0.059$, do not reject H_0
 (d) Yes, the equal variance assumption is valid (remember - this is a two sided test!)
- Reject H_0 since $q_3 = 8.33 > 7.815 = \chi_{0.05}^2(3)$
- Do not reject H_0 ($z = -1.32$ which is greater than $-z_{0.05} = -1.645$ so z is NOT in the critical region.)

5. Reject H_0 (Note the test statistic is $F = 6.0$)
6. (a) 51.90
(b) 58.10
(c) approximately 0.015
7. Do not accept H_0
8. (a) REJECTING/TRUE
(b) ACCEPTING/FALSE
9. (a) TRUE
(b) FALSE
(c) TRUE
10. I don't feel like drawing the picture here. I'm sure yours is lovely.
11. 0.4744
12. We discussed this in class. Explain it in your own words.