Math 365 Final Exam Review Sheet

- The final exam is Wednesday March 16 from 10-11:50am in MNB 104.
- The final is comprehensive and will cover Chapters $1, 2, 3$ (omit sections we did not cover).
- You may use your calculator, the front cover of your book, and any table in the back of the book.
- You may use one page of notes ($8\frac{1}{2}$ x 11 both sides).
- I will have office hours on Monday from 1-2pm and on Tuesday from 2-4pm. If you cannot come at these times and need help, please contact me and we will try to make an appointment at another time.
- Below are some sample problems (some you've seen before), but are not necessarily an exhaustive list of the type of questions that I will ask on the exam. I also recommend you go back and review your homework, exam, quizzes and other review sheets.

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1. Five cards are dealt at random from a standard 52 card deck. Find the probability of being dealt the following hands:
   (a) A full house (3 of the same of one value and 2 of a different value XXYYY).
   (b) A full house with exactly one diamond.
   (c) Two pairs (XXYYZ).
2. A deli offers a lunch special of 1 drink, 1 sandwich and 1 side for 5.95. The drinks to choose from are soda, coffee or lemonade; the sandwiches are veggie, peanut butter and jelly, or egg salad; the sides are fries, salad, chips, or fruit. How many different possible lunch specials are there?
3. Suppose A and B are independent events with $P(A) = .65$ and $P(B) = .1$ Compute:
   (a) $P(A \cap B)$
   (b) $P(A \cup B)$
   (c) $P(A' \cup B')$
   (d) $P(A \mid B)$
4. Prove that if A and B are independent, then so is A' and B.
5. A six sided red die has sides with the following: two 3s four 5s. A six sided white die has two 6s and four 4s. If both dice are tossed at random, what is the probability that the white die shows the higher number?
6. (a) A box contains 5 slips - 1 labeled win and 4 labeled loose. You and a friend take turns taking a slip of paper out of the box (without replacement). Should you go first or second to have the highest probability of selecting the "win" slip?
   (b) Suppose instead the box had 100 slips - 1 labeled win and 99 labeled loose. Now should you go first or second to have the highest probability of winning?
7. Find the mean and variance of the p.m.f. $f(x) = \frac{4-x}{6}$ $x = 1, 2, 3$.
8. On a 15 question multiple choice test there are 5 possible answers. Suppose a student guesses randomly on the test and let X be the number correct.
   (a) How is X distributed?
   (b) Find the Probability of getting at least 5 correct.
   (c) Find the Probability of getting 3 correct.

(d) Find the probability of getting 12 or fewer WRONG.

9. A super size bag of Grandma's mini-cookies contains 40 cookies where 15 are ginger snaps.

   (a) If 10 cookies are chosen at random find the probability that exactly 4 are ginger snaps.

   (b) If 10 cookies are chosen at random find the probability that exactly 4 are ginger snaps given that at least two are ginger snaps.

For the next two questions consider the following:
Suppose that people arrive at McDonalds at a mean rate of 5 per 10 minutes according to a Poisson process.

10. Let X be the number of people who arrive during a half hour period.

    (a) How is X distributed?

    (b) Find the $P(X \geq 25)$

11. Let X be the waiting time until the $4th$ customer of the day arrives.

    (a) How is X distributed?

    (b) Find the probability that $X \leq 13.36$ minutes.

12. On an electronic multiple choice test each question has 5 possible answers. Students answer questions until they get one right. Suppose a student guesses randomly on the test. Let X be the number of questions the student has to answer before they get one right.

    (a) How is X distributed?

    (b) Find the probability that $X \geq 10$.

13. Let X have moment generating function $M(t) = (1 - 2t)^{-5}$.

    (a) Find $E(X)$

    (b) Find $Var(X)$.

14. A theater sold advance tickets for an upcoming show. The theater can seat 295 people, but sells 300 tickets on the bet that not everyone will show up. If the probability that a given person won't show up is 0.02, then what is the probability that the theater can accomodate everyone who shows up?

15. A recent IRS report claimed that 60% of Americans make a mistake on their tax forms. Let X be the number of tax forms containing a mistake in a random sample of 20.

    (a) How is X distributed?

    (b) Find the mean of X.

    (c) Find the probability that $X = 15$.

    (d) Find the probability that $X \geq 12$.

16. Mary's Market sells eggs by the dozen. Unfortunately, each carton has a 15% chance of containing a cracked egg. Find the fewest number of cartons you must buy so that the probability of at least one carton containing a cracked egg is at least 0.90.

17. Molly's Market sells eggs by the dozen. Currently Molly has 25 cartons of eggs in stock of which 9 contain a cracked egg. If you buy 10 cartons of eggs, find the probability that 3 have a cracked egg.

18. Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from bowl B.

19. A life insurance company issues standard, preferred, and ultra-preferred policies. Of the company's policy holders of a certain age, 60% are standard with a probability of 0.01 of dying in the next year, 30% preferred with a probability of 0.008 of dying in th enext year, and 10% are ultra-preferred with a probability of 0.007 of dying in the next year. A policyholder of that age dies in the next year. What are the conditional probabilities of the deceased being standard, preferred, and ultra-preferred?

20. Let the random variable X be the number of days that a certain patient needs to be in the hospital. Suppose X has the p.m.f. $f(x) = \dfrac{5-x}{10}$, $x = 1, 2, 3, 4$. If the patient is to receive \$200 from an insurance company for each of the first two days in the hospital and \$100 for each day after the first tow days (up to 4 days), what is the expected paymet for the hospitalization?

21. Could the following data come from a binomial distribution $X \sim b(4, 0.6)$? Give several pieces of evidence to support your claim.

0	1	1	1	1
2	2	2	2	2
2	2	2	2	3
3	3	3	3	3
3	3	4	4	4

22. Let X equal the weight of the soap in a "6-pound" box. Assume that the distribution of X is $N(6.05, 0.0004)$.

    (a) Find $P(X < 6.0171)$.

    (b) If nine boxes of soap are selected at random from the production line, find the probability that at most two boxes weigh less than 6.0171 each.

23. Suppose $X \sim N(45, 100)$. Find the following probabilities:

    (a) $P(X \geq 57.3)$

    (b) $P(41 \leq X \leq 66)$

    (c) $P((X - 45)^2 \leq 384.1)$

24. A canymaker produces chocolate bars with a label weight of 2 ounces. Assume that the distribution of the weights of these chocolate bars is $N(1.98, 0.16)$. Find the probability that in a shipment of 20 bars there are 10 or more that weigh less than 1.77 ounces.

25. A company selling peanuts advertises its bags of peanuts as weighing 1.75 ounces. Suppose the weight of the peanut bags follows a normal distribution, $N(1.75, \sigma^2)$. What should σ be so that the probability that a randomly chosen bag of peanuts weighs less than 1.7 ounces is (approximately) 0.01?

Answers

1. (a) 0.00144
   (b) 0.00072
   (c) 0.0475
2. 36
3. (a) 0.065
   (b) 0.685
   (c) 0.935
   (d) 0.65
4. Prove it!
5. $\frac{5}{9}$
6. (a) You should go first.
   (b) It doesn't matter who goes first.
7. $\mu = \frac{5}{3}; \sigma^2 = \frac{5}{9}$
8. (a) $X \sim b(15, .2)$
   (b) 0.1642
   (c) 0.25
   (d) 0.602
9. (a) 0.285
   (b) 0.297
10. (a) X is Poisson with $\lambda = 15$
    (b) 0.011
11. (a) $X \sim \chi^2(8)$
    (b) 0.9
12. (a) Geometric with $p = 0.2$
    (b) 0.134
13. (a) 10
    (b) 20
14. 0.715 (Hint: Use Poisson approximation to Binomial)
15. (a) $X \sim b(20, 0.6)$
    (b) $\mu = 12$
    (c) 0.075
    (d) 0.5956
16. 15
17. 0.294
18. 0.575

19. P(Standard | Dies) = 0.659, P(Preferred | Dies) = 0.246, P(Ultra | Dies) = 0.077
20. \$360
21. Compare theoretical and experimental mean and variance; compare relative frequency of outcomes to theoretical probabilities, etc.
22. (a) 0.05
    (b) 0.9916
23. (a) 0.1093
    (b) 0.6375
    (c) 0.9500
24. 0.048
25. 0.0215