

Math 365 Exam 2 Review Problems

• Exam 2 will cover §2.4 – §2.6, and §3.1 – §3.5. You may use a 3 x 5 notecard (both sides) and all tables in the back of your book. You may also refer to the formulas in the front cover of your book.

<p>Let X be a discrete random variable. p.m.f. $f(x) = P(X = x)$ $F(x) = P(X \leq x) = \sum_{(k \in S, k \leq x)} f(x)$ $E[u(x)] = \sum_{x \in S} u(x)f(x)$ $\mu = E(X) = \sum_{x \in S} xf(x)$ $\sigma^2 = E((x - \mu)^2) = E(X^2) - \mu^2$ $M(t) = E(e^{tx})$ $M'(0) = \mu$ $M''(0) - \mu^2 = \sigma^2$</p>	<p>Let X be a continuous random variable. p.d.f. $f(x)$ $F(x) = P(X \leq x) = \int_{-\infty}^x f(x)dx$ $E[u(x)] = \int_{-\infty}^{\infty} u(x)f(x)dx$ $\mu = E(X) = \int_{-\infty}^{\infty} xf(x)dx$ $\sigma^2 = E((x - \mu)^2) = E(X^2) - \mu^2$ $M(t) = E(e^{tx})$ $M'(0) = \mu$ $M''(0) - \mu^2 = \sigma^2$ π_p is the (100pth)percentile if $F(\pi_p) = \int_{-\infty}^{\pi_p} f(x)dx = p$</p>
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1. If X has the given m.g.f. below, then (i) How is X distributed? (ii) Find μ and σ^2 .

(a) $M(t) = \sum_1^5 (.2)e^{tx}$

(b) $M(t) = \frac{.1e^t}{1 - .9e^t}$

(c) (Don't worry about this one) $M(t) = e^{50t+800t^2}$

2. Suppose X is a continuous R.V. with pdf $f(x) = e^{-x-1}$, $-1 < x < \infty$.

(a) Find $M(t)$

(b) Find μ

(c) Find σ^2

(d) Find $F(x)$

(e) Find $P(X \leq 2)$

(f) Find the 25th percentile, $\pi_{.25}$.

3. Burger City is having a promotion where it passes out scratch tickets. Each ticket has a 10% chance of a win. Suppose you get 25 tickets and X is the number of winning tickets.

(a) What is the distribution of X ?

(b) What is $E[X]$?

(c) What is $P(X = 3)$?

(d) What is $P(X = 11)$?

(e) How many tickets should you buy to expect to have about 7 winning tickets?

4. A busy tutoring center is open evenings from 5 pm to 8 pm. Suppose students seeking help with their probability homework come in on average 3 every 12 minutes according to a Poisson process.
 - (a) How many students are expected to come in seeking help on their probability homework in a typical evening at the tutoring center?
 - (b) What is the probability that 10 or more students seeking help with probability homework come during the first half hour that the center is open?
 - (c) What is the probability that the first student to arrive comes in the first 5 minutes?
 - (d) What is the probability that more than 10 minutes pass before the 3rd student arrives?

5. An airline always overbooks, if possible. A particular flight with only 15 seats sells tickets for \$400 each. The airline sells 20 seats for the flight.
 - (a) If the probability of a passenger not showing up for the flight is 0.45, then assuming independence, what is the probability that the airline can accommodate all the passengers who do show up?
 - (b) If the airline must return the \$400 price plus a penalty of \$500 to each passenger that cannot get on the flight, what is the expected penalty that the airline will pay?

6. Let $f(x) = \frac{4}{9}x^2$, $-c \leq x \leq c$. Find the value of the constant c that makes $f(x)$ the p.d.f. of a random variable X .

7. Suppose that $X \sim b(20, .6)$
 - (a) Find $P(X \leq 5)$
 - (b) Find $P(X > 11)$
 - (c) Find $P(X = 7)$

8. Let $f(x) = \frac{1}{2}e^{-x/2}$, $0 \leq x < \infty$ be the p.d.f. of a random variable X . Find μ .

9. Suppose that X has a Chi-Square distribution with 8 degrees of freedom. Find a so that $P(X \leq a) = 0.05$.

10. Suppose the probability of an item being defective is 0.01. In a shipment of 1000 items, what is the probability that more than 14 are defective? (Hint: Use Poisson approx. to Binomial.)

11. Mary is studying to be a mathematician. To pay for college, she works the graveyard shift at a toll booth. Mary notices that cars arrive at her toll booth at the rate of 2 cars every 10 minutes according to a Poisson process. Use this information to answer the following questions.
 - (a) If Mary works an 8 hour shift, how many cars does she expect to pass through her booth?
 - (b) What is the probability that Mary will have to wait more than 8 minutes before the first car arrives after she starts her shift?
 - (c) What is the probability that Mary will have to wait no more than 10 minutes before 3 cars arrive at her booth?
 - (d) What is the probability that Mary will see 15 or more cars in a given hour of her shift?
 - (e) What is the probability that during Mary's 8 hour shift, during exactly 3 of her 8 hours she will see 15 or more cars (i.e., between 11-12 or 12-1 or 1-2, etc)?

12. The time (in seconds) for a second grader to bounce a ball 10 times was recorded for a class of 30 second graders and is given below:

5.5	6.2	10.5	12.7	10.3	8.2
4.9	6.8	13.5	7.3	9.1	7.2
5.1	5.9	20.4	5.9	25	8.9
6.5	6.0	12.2	8.8	15	30
15.2	7.2	9.1	9.7	11	4.2

- (a) Make a stem and leaf plot of this data.
- (b) Give the 5 number summary of this data.
- (c) Make a box and whisker plot for this data.
- (d) Are there any outliers? Explain how you know.