Math 365

# Chapter 1 Practice Problems WITH ANSWERS

1. At one point in the 2008 US Presidential Campaign there were 7 candidates from the Republican or Democrat party still in the running:
   Democrat Party Candidates: Clinton, Obama, Gravel
   Republican Party Candidates: McCain, Romney, Huckabee, Paul

   (a) How many different ways can the candidates be lined up?

   5040

   (b) How many different ways can the candidates be lined up so that no one is standing next to someone else of the same party (no Republicans next to each other and no Democrats next to each other)?

   144

   (c) Selecting only from the list of Republican candidates, how many ways are there to choose a President and a Vice President?

   12

   (d) Suppose three candidates are selected at random from the seven above. Find the probability that exactly two of them are from the same party.

   0.857

   (e) The seven candidates are being chosen randomly for a game of dodgeball. What is the probability that the 3rd person chosen is a Republican?

   0.5714

   (f) The seven candidates are being chosen randomly for a game of dodgeball. What is the probability that the 3rd person chosen is a Republican given that the first person chosen was a Democrat?

   0.467

2. DON'T DO THIS ONE. A fair coin is tossed repeatedly 10 times. Let $X$ be the number of tosses until two heads appear in a row (the first time). Find the p.m.f. of $X$.

3. An urn contains 4 red balls, 6 red cubes, 8 blue balls, and some blue cubes as well. An object is selected at random from the urn. Let $A$ be the event that the object is a ball and let $B$ be the event that the object is blue. How many blue cubes must be placed in the urn so that $A$ and $B$ are independent events? Explain. (Hint: you need to find values so that $P(A \cap B) = P(A) \cdot P(B)$.)

   12 blue cubes.

4. The state of Oregon has some extra money from a gas tax that it would like to give back to drivers. Assume that license plates are assigned randomly to have the form
   LETTER LETTER LETTER NUMBER NUMBER NUMBER (repeats are allowed)

   The state will select a LETTER LETTER LETTER NUMBER NUMBER NUMBER and award money based on the following :

| Match this EXACTLY | WIN this amount |
|---|---|
| 3 letters AND 0 numbers | $\$1,000$ |
| 0 letters AND 3 numbers | \$750 |
| 2 letters AND 0 numbers | \$500 |
| 0 letters AND 2 numbers | \$100 |
| 1 letters AND 1 numbers | \$50 |
| Otherwise | 0 |

   (NOTE: Order matters, you must match the letters and the position they are in; i.e. if the randomly selected license plate is XYZ123 and yours is XZP213 you have matched 1 letter (X) and one number (3)) Let $X$ be the winnings of a randomly chosen license plate. Find the p.m.f. of $X$.

   $f(1000) = 0.0000415$
   $f(750) = 0.00089$
   $f(500) = 0.0031$
   $f(100) = 0.024$
   $f(50) = 0.026$
   $f(0) = 0.9459685$

5. During a blackout 100 people are arrested on suspicion of looting. Each person takes a polygraph test. Suppose it is known that the results of the polygraph are 90% reliable when administered to a guilty person and 98% reliable when given to an innocent person. Further suppose we know that the probability that any given suspect is guilty is 0.12. A random suspect is tested and the polygraph result says that the person is guilty. Find the probability that the suspect is actually innocent.

   0.14