Math 365 Exam 1 Review Problems

• Exam 1 is an in-class exam given on Friday February 5, 2010.
• Exam 1 covers Chapter 1 and Chapter 2 Sections 1 – 4.
• During the exam you may use one 3x5 notecard with notes on both sides, your calculator, and the Binomial table in the back of the book.

1. Suppose you have the following information: $A, B, C$ are subsets of $S$ and

   • $P(A) = .3$
   • $P(B') = .375$
   • $P(A \cap B) = .1875$
   • $P(B \cap C) = .1$
   • $P(C) = .2$

Find the following probabilities:

(a) $P(A \cup B)$
(b) $P(C \mid B)$
(c) $P(C' \mid B)$
(d) Are $C$ and $B$ independent? Explain.
(e) Are $A$ and $B$ independent? Explain.
(f) Are $A$ and $B$ mutually exclusive? Explain.

2. A display case contains 35 gems, of which 10 are real diamonds and 25 are fake diamonds. Four burglars come in one at a time and randomly steal one gem each (without replacement, of course).

   (a) What is the probability that the second burglar steals a real diamond?
   (b) What is the probability that at least one real diamond is stolen?
   (c) What is the probability that the fourth burglar steals a fake diamond given that the first three burglars each took one real diamond?

3. A random 5–card poker hand is dealt from a standard 52 card deck. What is the probability that the hand contains exactly 1 club and a three of a kind? (Hint: the club can be part of the three of a kind, but we DON’T want a 4 of a kind or a full house (which is a 3 of a kind and a pair).

4. In the Lucky 3 lottery, 50 balls numbered 1 – 50 are placed in an urn. Three balls are randomly selected WITH replacement. If a player matches one ball, they win $25, two balls wins $1,000, and all three wins $50,000. Suppose that a lottery ticket costs $1. Let $X$ be the net winnings of the game. Find $E(X)$.

5. An airline always overbooks, if possible. A particular flight with only 15 seats sells tickets for $400 each. The airline sells 20 seats for the flight.

   (a) If the probability of a passenger not showing up for the flight is 0.45, then assuming independence, what is the probability that the airline can accomodate all the passengers who do show up?
(b) If the airline must return the $400 price plus a penalty of $500 to each passenger that cannot get on the flight, what is the expected penalty that the airline will pay?

6. In the Early Learners Pre-School, 67% of the children have been vaccinated for chicken pox. Suppose that the probability that a vaccinated child gets the chicken pox if 0.05 and the probability that an unvaccinated child gets the chicken pox is 0.55. Suppose that one of the children in the Pre-School has the chicken pox. What is the probability that this child was vaccinated?

7. Suppose that $A$ and $B$ are independent events. Then prove that $A'$ and $B$ are also independent.

8. Using only the properties of probability (p. 16 in your book), prove that if $A \subset B$, then $P(A) \leq P(B)$.

9. Burger City is having a promotion where it passes out scratch tickets. Each ticket has a 10% chance of a win. Suppose you get 25 tickets and $X$ is the number of winning tickets.

   (a) What is the distribution of $X$?
   (b) What is $E[X]$?
   (c) What is $P(X = 3)$?
   (d) What is $P(X = 11)$?
   (e) How many tickets should you buy to expect to have about 7 winning tickets?

10. In a pile of 100 scratch tickets from Burger Planet, 10 of them are winners. Select a handful of 25 of these tickets at random and let $X$ be the number of winning tickets.

    (a) What is the distribution of $X$?
    (b) What is $E[X]$?
    (c) What is $P(X = 3)$?
    (d) What is $P(X = 11)$?

11. Suppose that $X \sim b(15, .25)$

    (a) Find $P(X \geq 1)$
    (b) Find $P(X < 4)$
    (c) Find $P(X = 4)$

12. Suppose that $X \sim b(20, .6)$

    (a) Find $P(X \leq 5)$
    (b) Find $P(X > 11)$
    (c) Find $P(X = 7)$

13. An urn contains red and blue balls. There are 50 balls total. If five balls are selected at random, how many of the 50 balls should be blue so that the expected number of blue balls in this experiment is 3.5?

14. Suppose it is known that the probability that a WOU student will watch the Super Bowl is 0.65. If 20 WOU students are selected at random, find the probability that:

    (a) At most 12 will watch the Super Bowl.
    (b) Exactly 15 will watch the Super Bowl.
    (c) More than 12 will NOT watch the Super Bowl.