Math 365 Final Exam Review Sheet

- The final exam is Wednesday March 18 from 10am - 12 noon in MNB 110.
- The final is comprehensive and will cover Chapters $1, 2, 3$, §4.1, §4.2, §5.2, and §5.3.
- You may use your calculator, the front cover of your book, and any table in the back of the book.
- You may use one page of notes ($8\frac{1}{2}$ x 11 both sides).
- Go back and review your homework problems, exams, and quizzes.
- I will have office hours on Tuesday from 1-3pm and Wed. from 9-10am.
- Below are some sample problems, but are not meant to be an exhaustive list of the type of questions that I will ask on the exam.

1. Five cards are dealt at random from a standard 52 card deck. Find the probability of being dealt the following hands:
   (a) A three of a kind (3 of the same value and 2 different)
   (b) A full house (3 of the same of one value and 2 of a different value).

2. A deli offers a lunch special of 1 drink, 1 sandwich and 1 side for 5.95. The drinks to choose from are soda, coffee or lemonade; the sandwiches are veggie, peanut butter and jelly, or egg salad; the sides are fries, salad, chips, or fruit. How many different possible lunch specials are there?

3. Suppose A and B are independent events with $P(A) = .65$ and $P(B) = .1$ Compute:
   (a) $P(A \cap B)$
   (b) $P(A \cup B)$
   (c) $P(A' \cup B')$
   (d) $P(A \mid B)$

4. Prove that if A and B are independent, then so is A' and B.

5. A six sided red die has sides with the following: two 3s four 5s. A six sided white die has two 6s and four 4s. If both dice are tossed at random, what is the probability that the white die shows the higher number?

6. (a) A box contains 5 slips - 1 labeled win and 4 labeled loose. You and a friend take turns taking a slip of paper out of the box (without replacement). Should you go first or second to have the highest probability of selecting the "win" slip?

   (b) Suppose instead the box had 100 slips - 1 labeled win and 99 labeled loose. Now should you go first or second to have the highest probability of winning?

7. Find the mean and variance of the p.m.f. $f(x) = \frac{4-x}{6}$ $x = 1, 2, 3$.

8. On a 15 question multiple choice test there are 5 possible answers. Suppose a student guesses randomly on the test and let X be the number correct.
   (a) How is X distributed?

(b) Find the Probability of getting at least 5 correct.
(c) Find the Probability of getting 3 correct.
(d) Find the probability of getting 12 or fewer WRONG.

9. A super size bag of Grandma's mini-cookies contains 40 cookies where 15 are ginger snaps. If 10 cookies are chosen at random find the probability that 4 are ginger snaps.

For the next two questions consider the following:
Suppose that people arrive at McDonalds at a mean rate of 5 per 10 minutes according to a Poisson process.

10. Let X be the number of people who arrive during a half hour period.

    (a) How is X distributed?
    (b) Find the $P(X \geq 25)$

11. Let X be the waiting time until the $4th$ customer of the day arrives.

    (a) How is X distributed?
    (b) Find the probability that $X \leq 13.36$ minutes.

12. On an electronic multiple choice test each question has 5 possible answers. Students answer questions until they get one right. Suppose a student guesses randomly on the test. Let X be the number of questions the student has to answer before they get one right.

    (a) How is X distributed?
    (b) Find the probability that $X \geq 10$.

13. At the Super Mall, it is known that 20% of customers use cash, 65% use Visa, and 15% use Mastercard. Select 3 customers at random and let X be the number who pay in cash and Y the number who pay with a Visa.

    (a) Find the joint p.m.f. of X and Y.
    (b) Find $P(X = 1, Y = 2)$
    (c) Find $Cov(X, Y)$
    (d) Find the correlation coefficient ρ.
    (e) Find the equation of the least squares regression line.

14. Let X and Y have joint p.m.f. $f(x, y) = 2e^{-x-y},\ 0 \leq x \leq y < \infty$. Find the marginal p.d.f.s of X and Y, $f_1(x)$ and $f_2(y)$. Are X and Y independent?

15. Let X have moment generating function $M(t) = (1 - 2t)^{-5}$.

    (a) Find $E(X)$
    (b) Find $Var(X)$.

16. A theater sold advance tickets for an upcoming show. The theater can seat 295 people, but sells 300 tickets on the bet that not everyone will show up. If the probability that a given person won't show up is 0.02, then what is the probability that the theater can accomodate everyone who shows up?

17. Let X have an exponential distribution with $\theta = 2$. Let $Y = e^X$ be a random variable. Find the p.d.f. of Y.

18. A recent IRS report claimed that 60% of Americans make a mistake on their tax forms. Let X be the number of tax forms containing a mistake in a random sample of 20.

    (a) How is X distributed?

    (b) Find the mean of X.

    (c) Find the probability that $X = 15$.

    (d) Find the probability that $X \geq 12$.

19. Mary's Market sells eggs by the dozen. Unfortunately, each carton has a 15% chance of containing a cracked egg. Find the fewest number of cartons you must buy so that the probability of at least one carton containing a cracked egg is at least 0.90.

20. Molly's Market sells eggs by the dozen. Currently Molly has 25 cartons of eggs in stock of which 9 contain a cracked egg. If you buy 10 cartons of eggs, find the probability that 3 have a cracked egg.

21. Bowl A contains three red and two white chips, and bowl B contains four red and three white chips. A chip is drawn at random from bowl A and transferred to bowl B. Compute the probability of then drawing a red chip from bowl B.

22. A life insurance company issues standard, preferred, and ultra-preferred policies. Of the company's policy holders of a certain age, 60% are standard with a probability of 0.01 of dying in the next year, 30% preferred with a probability of 0.008 of dying in th enext year, and 10% are ultra-preferred with a probability of 0.007 of dying in the next year. A policyholder of that age dies in the next year. What are the conditional probabilities of the deceased being standard, preferred, and ultra-preferred?

23. Let $\overline{X}$ be the average of $n = 50$ random samples from a distribution with $\mu = 2.4$, $\sigma^2 = 4$. Find $P(\overline{X} \geq 3)$.

24. Let X be the number of miles students at Green Leaf Community College commute per day. Suppose X is Normally distributed with $\mu_X = 20$, $\sigma_X = 5$. In a random sample of 10 students, find the probability that the combined total number of miles driven is greater than 225.

25. Let X and Y equal the respective number of hours a randomly selected woman or man spends watching t.v. per week. Suppose that it is known that X is $N(12, 100)$ and Y is $N(10, 225)$. If a man and woman are selected at random, find the probability that the woman watches more t.v. than the man.

26. Let X equal the weight of the soap in a "6-pound" box. Assume that the distribution of X is $N(6.05, 0.0004)$.

    (a) Find $P(X < 6.0171)$.

    (b) If nine boxes of soap are selected at random from the production line, find the probability that at most two boxes weigh less than 6.0171 each. Hint: Let Y equal the number of boxes that weigh less than 6.0171.

    (c) Let $\overline{X}$ be the sample mean of the nine boxes. Find $P(\overline{X} \leq 6.035)$.

27. For Company A, there is an 80% chance that no claims are made; but if one or more claims are made, the total claims has a normal distribution with mean $20,000$ and standard deviation $5,000$. For Company B, there is a 90% chance of no claims; but if one or more claims are made, the total claims has a normal distribution with mean $30,000$ and standard deviation $6,000$. Assuming independence, what is the probability that $B's$ total claims exceed those of A?

Answers

1. (a) 0.02113
   (b) 0.00144
2. 36
3. (a) 0.065
   (b) 0.685
   (c) 0.935
   (d) 0.65
4. Prove it!
5. $\frac{5}{9}$
6. (a) You should go first.
   (b) It doesn't matter who goes first.
7. $\mu = \frac{5}{3}; \sigma^2 = \frac{5}{9}$
8. (a) $X \sim b(15, .2)$
   (b) 0.1642
   (c) 0.25
   (d) 0.602
9. 0.285
10. (a) X is Poisson with $\lambda = 15$
    (b) 0.011
11. (a) $X \sim \chi^2(8)$
    (b) 0.9
12. (a) Geometric with $p = 0.2$
    (b) 0.134
13. (a) $f(x, y) = \frac{3!}{x!y!(3-x-y)!}(0.2)^x(0.65)^y(0.15)^{3-x-y}$
    (b) 0.2535
    (c) -0.39
    (d) -0.681
    (e) $y = 2.44 - 0.812x$
14. $f_1(x) = 2e^{-2x}$; $f_2(y) = -2e^{-2y} + 2e^{-y}$, dependent
15. (a) 10
    (b) 20
16. 0.715 (Hint: Use Poisson approximation to Binomial)
17. $g(y) = \frac{1}{2y^{3/2}}, \; 1 \le Y < \infty$

18. (a) $X \sim b(20, 0.6)$
    (b) $\mu = 12$
    (c) 0.075
    (d) 0.5956

19. 15

20. 0.294

21. 0.575

22. P(Standard | Dies) = 0.659, P(Preferred | Dies) = 0.246, P(Ultra | Dies) = 0.077

23. 0.0170

24. 0.0571

25. 0.5438

26. (a) 0.05
    (b) 0.9916
    (c) 0.0122

27. 0.098