

## Math 365 Exam 2 Review Problems

• Exam 2 will cover §2.5 – §2.6, §3.1 – §3.5 and §4.1. You may use a full page of notes (front and back) and all tables in the back of your book. You may also refer to the formulas in the front cover of your book.

1. If  $X$  has the given m.g.f. below, then (i) How is  $X$  distributed? (ii) Find  $\mu$  and  $\sigma^2$ .

(a)  $M(t) = \sum_1^5 (.2)e^{tx}$

(b)  $M(t) = \frac{.1e^t}{1 - .9e^t}$

2. Suppose  $X$  is a continuous R.V. with pdf  $f(x) = e^{-x-1}$ ,  $-1 < x < \infty$ .

- (a) Find  $M(t)$
- (b) Find  $\mu$
- (c) Find  $\sigma^2$
- (d) Find  $F(x)$
- (e) Find  $P(X \leq 2)$
- (f) Find the 25th percentile,  $\pi_{25}$ .

3. A busy tutoring center is open evenings from 5 pm to 8 pm. Suppose students seeking help with their probability homework come in on average 3 every 12 minutes according to a Poisson process.

- (a) How many students are expected to come in seeking help on their probability homework in a typical evening at the tutoring center?
- (b) What is the probability that 10 or more students seeking help with probability homework come during the first half hour that the center is open?
- (c) What is the probability that the first student to arrive comes in the first 5 minutes?
- (d) What is the probability that more than 10 minutes pass before the 3rd student arrives?

4. Suppose  $X$  has p.d.f.  $f(x) = \frac{1}{2}e^{-x/2}$ ,  $0 \leq x < \infty$ . Let  $Y = X^2$ , then find the p.d.f. for  $Y$ .

5. Let  $f(x) = \frac{4}{9}x^2$ ,  $-c \leq x \leq c$ . Find the value of the constant  $c$  that makes  $f(x)$  the p.d.f. of a random variable  $X$ .

6. Let  $f(x) = \frac{1}{2}e^{-x/2}$ ,  $0 \leq x < \infty$  be the p.d.f. of a random variable  $X$ . Find  $\mu$ .

7. Suppose that  $X$  has a Chi-Square distribution with 8 degrees of freedom. Find  $a$  so that  $P(X \leq a) = 0.05$ .

8. Suppose the probability of an item being defective is 0.01. In a shipment of 1000 items, what is the probability that more than 14 are defective? (Hint: Use Poisson approx. to Binomial.)

9. Mary is studying to be a mathematician. To pay for college, she works the graveyard shift at a toll booth. Mary notices that cars arrive at her toll booth at the rate of 2 cars every 10 minutes according to a Poisson process. Use this information to answer the following questions.
- (a) If Mary works an 8 hour shift, how many cars does she expect to pass through her booth?
  - (b) What is the probability that Mary will have to wait more than 8 minutes before the first car arrives after she starts her shift?
  - (c) What is the probability that Mary will have to wait no more than 10 minutes before 3 cars arrive at her booth?
  - (d) What is the probability that Mary will see 15 or more cars in a given hour of her shift?
  - (e) What is the probability that during Mary's 8 hour shift, during exactly 3 of her hours she will see 15 or more cars (i.e., between 11-12 or 12-1 or 1-2, etc)?

10. Let  $X$  and  $Y$  have joint p.d.f. given by

$$f(x, y) = 8xy, \quad 0 \leq x \leq y \leq 1$$

- (a) Find the marginal p.d.f. of  $Y$ .
  - (b) Find  $\mu_Y$  using the joint p.d.f..
  - (c) Find  $\mu_Y$  using the marginal p.d.f. of  $Y$ .
  - (d) Find  $P(X < 1/2)$ .
  - (e) Are  $X$  and  $Y$  independent? Explain.
11. Phone calls arrive at a hotline on average of 2 per 10 minutes. Lazy Larry works a half hour shift at the hotline 5 days a week. Find the expected number of times during his 5 day work week that Larry will have to wait more than 8 minutes to answer the first call of his shift.