

Math 365 Exam 1 Review Problems

1. Suppose you have the following information: A, B, C are subsets of S and

- $P(A) = .3$
- $P(B') = .375$
- $P(A \cap B) = .1875$
- $P(B \cap C) = .1$
- $P(C) = .2$

Find the following probabilities:

- $P(A \cup B)$
 - $P(C \mid B)$
 - $P(C' \mid B)$
 - Are C and B independent? Explain.
 - Are A and B independent? Explain.
 - Are A and B mutually exclusive? Explain.
2. A display case contains 35 gems, of which 10 are real diamonds and 25 are fake diamonds. Four burglars come in one at a time and randomly steal one gem each (without replacement, of course).
- What is the probability that the second burglar steals a real diamond?
 - What is the probability that at least one real diamond is stolen?
 - What is the probability that the fourth burglar steals a fake diamond given that the first three burglars each took one real diamond?
3. A random 5-card poker hand is dealt from a standard 52 card deck. What is the probability that the hand contains exactly 1 club and a three of a kind? (Hint: the club can be part of the three of a kind, but we DON'T want a 4 of a kind or a full house (which is a 3 of a kind and a pair).
4. In the Lucky 3 lottery, 50 balls numbered 1–50 are placed in an urn. Three balls are randomly selected WITH replacement. If a player matches one ball, they win \$25, two balls wins \$1,000, and all three wins \$50,000. Suppose that a lottery ticket costs \$1. Let X be the net winnings of the game. Find $E(X)$.
5. An airline always overbooks, if possible. A particular flight with only 15 seats sells tickets for \$400 each. The airline sells 20 seats for the flight.
- If the probability of a passenger not showing up for the flight is 0.45, then assuming independence, what is the probability that the airline can accommodate all the passengers who do show up?
 - If the airline must return the \$400 price plus a penalty of \$500 to each passenger that cannot get on the flight, what is the expected penalty that the airline will pay?
6. In the Early Learners Pre-School, 67% of the children have been vaccinated for chicken pox. Suppose that the probability that a vaccinated child gets the chicken pox is 0.05 and the probability that an unvaccinated child gets the chicken pox is 0.55. Suppose that one of the children in the Pre-School has the chicken pox. What is the probability that this child was vaccinated?

7. Suppose that A and B are independent events. Then prove that A' and B are also independent.
8. Using only the properties of probability (p. 16 in your book), Prove that if $A \subset B$, then $P(A) \leq P(B)$.
9. Burger City is having a promotion where it passes out scratch tickets. Each ticket has a 10% chance of a win. Suppose you get 25 tickets and X is the number of winning tickets.
 - (a) What is the distribution of X ?
 - (b) What is $E[X]$?
 - (c) What is $P(X = 3)$?
 - (d) What is $P(X = 11)$?
10. In a pile of 100 scratch tickets from Burger Planet, 10 of them are winners. Select a handful of 25 of these tickets at random and let X be the number of winning tickets.
 - (a) What is the distribution of X ?
 - (b) What is $E[X]$?
 - (c) What is $P(X = 3)$?
 - (d) What is $P(X = 11)$?
11. Suppose that $X \sim b(15, .25)$
 - (a) Find $P(X \geq 1)$
 - (b) Find $P(X < 4)$
 - (c) Find $P(X = 4)$
12. Suppose that $X \sim b(20, .6)$
 - (a) Find $P(X \leq 5)$
 - (b) Find $P(X > 11)$
 - (c) Find $P(X = 7)$