Math 346 Final Review Problems

The final exam for MTH 346 is Friday, March 21, 8-9:50am in **MNB 110**. You may bring one page (usual size) of notes with notes on both sides, again with a maximum of 3 worked out problems or proofs. The following practice problems are meant to help you review, but are not an exhaustive list of all types of questions that could be on the final. It is also a good idea to review old homework, lab conjectures and proof, exams and exam review sheets.

Finals week office hours: M: 11-12; T: 3:30-5:00 (mainly for MTH 495/595 students); W: 12-2; R: 12-2

1. (a) TRUE FALSE If $n \mid ab$ and $n \not\mid a$, then $n \mid b$.

   (b) TRUE FALSE $6^4 7^3$ has exactly 20 divisors.

   (c) TRUE FALSE If $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$ then $a \equiv c \pmod{n}$.

   (d) TRUE FALSE There exists infinitely many integer solutions to $14x + 21y = 16$.

   (e) TRUE FALSE The additive order of 6 modulo 12 is 4.

2. Show that the square of every odd integer is of the form $8k + 1$.

3. Let p be a prime $p > 5$. What are the possible values $0 \leq a < 30$ so that $p \equiv a \pmod{30}$. Prove why these are the correct values.

4. Let p be prime, then prove that $a > 0$ is its own inverse modulo p if and only if $a \equiv 1 \pmod{p}$ or $a \equiv -1 \pmod{p}$.

5. If $gcd(a, n) = 1$, show that every integer is a multiple of $a \pmod{n}$.

6. Prove or Disprove: If $a \mid c$ and $b \mid c$ then $ab \mid c$.

7. Let x be a positive integer. Show that x is divisible by 9 if and only if the sum of the digits of x is divisible by 9.

8. Use the Chinese Remainder Theorem to find ALL solutions ($0 \leq x < 77$) to the congruence $x^2 - 15x - 43 \equiv 0 \pmod{77}$. Show work. No CRT, no credit.

9. Use the Euclidean Algorithm to find the inverse of 79 modulo 385. Show all work.

10. Show that $a^{120} - 1$ is divisible by 77 whenever $gcd(a, 77) = 1$.

11. Solve the following pairs of congruences or explain why no solution is possible. Give answers x so that $x \geq 0$.

    (a) $x \equiv 2420 \pmod{5445}$
    $x \equiv 2750 \pmod{4125}$

    (b) $x \equiv 2425 \pmod{5445}$
    $x \equiv 2750 \pmod{4125}$

12. Assume that p and q are distinct odd primes such that $p - 1 \mid q - 1$. If $gcd(a, pq) = 1$, show that $a^{q-1} \equiv 1 \pmod{pq}$.

13. Find $2398247502347^2 \pmod{10}$.

14. Find the smallest integer n, such that $d(n) = 15$.

15. Find a solution to $72 * m \equiv 0 \pmod{99}$.

16. Find a solution to $15x + 37y = 5642$ where x and y are both positive integers.

17. If $a, b, c \in \mathbf{Z}$, with $c \neq 0$, show that $a \mid b$ if and only if $ac \mid bc$.

18. If $p > 5$ is prime, prove that p is NOT congruent to 3 modulo 12.

19. Prove that if $gcd(a, b) = 1$, then $gcd(a + b, a - b) = 1$ or 2.

20. Prove that if $a, b, c, d \in \mathbf{Z}$ with $gcd(a, b) = 1$, $gcd(c, d) = 1$, and $\frac{a}{b} + \frac{c}{d} \in \mathbf{Z}$, then $b = d$.

21. Prove or disprove: If $a^5 \mid b^3$, then $a \mid b$.

22. Prove or disprove: If $a^3 \mid b^5$, then $a \mid b$.

23. Prove or disprove: $gcd(ma, mb) = mgcd(a, b)$.

24. (a) Find the remainders when 2^{50} and 41^{65} are divided by 7.
    (b) What is the remainder when the following sum is divided by 4?

$$1^5 + 2^5 + 3^5 + \cdots 99^5 + 100^5$$

25. Find all distinct congruence classes modulo 70 (i.e., all numbers less than 70) that have additive order 14.

26. Prove or Disprove: If $a \mid c$ and $b \mid c$ and $gcd(a, b) = 1$, then $ab \mid c$.

27. Use the Chinese Remainder Theorem to find the form of all solutions to the following set of congruences:

$$x \equiv 3 \pmod{12}$$

$$x \equiv 5 \pmod{7}$$

$$x \equiv 6 \pmod{19}$$

$$x \equiv 7 \pmod{23}$$