

Tuesday Questions Week 2

1. Show that $\gcd(a, b) = \gcd(a, b + na)$ for all integers n .
2. What are the possible values of $\gcd(a, a + 2)$?
3. What are the possible values of $\gcd(a, a + 3)$?
4. What are the possible values of $\gcd(a, a + 4)$?
5. Prove that $2 \mid n^2 - n$ for all positive integers n .
6. Prove or disprove: If $a^3 \mid b^2$ then $a \mid b$.
7. Prove or disprove: If $n \mid ab$ and $\gcd(n, a) = 1$, then $n \mid b$ (this is one of your homework problems).
8. Prove that if a, b, c, d are integers with $\gcd(a, b) = 1$, $\gcd(c, d) = 1$, and $\frac{a}{b} + \frac{c}{d}$ is an integer, then $b = d$.