Tuesday Problems Week 1

In your groups, solve as many problems as you can. You do not need to go in order. We will spend the last 15-20 minutes of class presenting results.

1. Suppose n is an integer with factorization:
   $n = p_1^{e_1} p_2^{e_2} \ldots p_k^{e_k}$ where p_i i=1..k are distinct prime numbers and e_i are positive integers. Prove that n is a square if and only if e_i is even for i=1..k

2. Prove that if x is an integer that is divisible by 3 then x^n is also divisible by 3 for n a positive integer.

3. Prove that if x and y are both integers that are divisible by 3 then x + y is also divisible by 3.

4. Consider the equation from the Simpson's claiming to be a counterexample to Fermat's Last Theorem:
   - $3987^{12} + 4365^{12} = 4472^{12}$ Give a mathematical argument explaining why this equation cannot be true

5. Show that any integer of the form 6k+5 is also of the form 3j+2, but not conversely (k,j $\in$ Z)

6. Use the division algorithm to show the following:
   A. Any integer can be represented by one of the following forms: 4q, 4q+1, 4q+2, or 4q+3 (q $\in$ Z)
   B. The square of any integer is either of the form 3k or 3k+1 (k$\in$ Z).
   C. The cube of any integer has one of the forms 9k, 9k+1 or 9k+8 (k $\in$ Z)
   D. The cube of any integer has one of the forms 7k, 7k+1 or 7k-1 (k$\in$ Z)
   E. Note that C&D don't contradict each other. Write 3^3 = 27 in one of the forms given in C and D.

7. Prove that no integer in the following sequence is a perfect square:
   11, 111, 1111, 11111, 111111,….
   - Hint:
     A. Prove that any number ending in 08 is divisible by 4 (e.g. 108, 3408,1208, 1108).
     B. Prove that the square of any integer has the from 4q or 4q+1 (q $\in$ Z).
     C. Re-write the numbers in the sequence in a convenient way.

8. Prove that there are an infinite number of prime numbers.