- Exam 2 is an in class given on Tues. 6/4 and covers chapters 14, 15, and 16.
- You may use one sheet of notes (one side, regular size paper, at most 3 worked out examples or proofs).
- Study ideas: work through the problems below, old homework, worksheets, etc. Time yourself so you can do problems quickly.

1. What is the remainder when $x^{35} - 1$ is divided by $x - 2$ in $\mathbb{Z}_5$.

2. In $\mathbb{Z}_7[x]$ write $x^3 + 1$ as a product of linear factors.

3. Suppose that $f(x) \in Z_2[x]$ and is irreducible over Z_2. What can you say about the number of nonzero coefficient in $f(x)$?

4. Find all odd primes p for which $x - 2$ is a factor of $x^4 + x^3 + x^2 + x$ in $Z_p[x]$.

5. Suppose $\phi : \mathbb{Z}_{20} \to \mathbb{Z}_{15}$ is a mapping defined by $\phi(x) = 6x$

(a) Prove that the mapping ϕ is a ring homomorphism.

(b) What is the image of ϕ?

(c) What is $Ker\phi$?

(d) Let $R = \mathbb{Z}_{20}/Ker(\phi)$. Write down the elements of R.

(e) Let R be defined as in the previous. Prove or disprove that R is a field.

(f) Give an example of a (different) mapping from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{15}$ of the form $\phi(x) = nx$, that is a ring homomorphism. Explain.

(g) Give an example of a (different) mapping from $\mathbb{Z}_{20}$ to $\mathbb{Z}_{15}$ of the form $\phi(x) = nx$, that is not a ring homomorphism. Explain.

6. Let $R = \mathbb{Z} \oplus \mathbb{Z}$. Prove that the ideal $I = \{(a, 0) \mid a \in \mathbb{Z}\}$ is prime but not maximal (you do not need to prove it is an ideal).

7. Prove or disprove: $Z_2[x]/\langle x^3 + 1\rangle$ is a field.

8. Suppose that R is a ring with unity and the mapping $\phi(r) = -r$ is a ring homomorphism from R to R. What can you say about the characteristic of R? Justify your answer.

9. Let R and S be rings with respective unities 1_R and 1_S. If $\phi : R \to S$ is a ring homomorphism from R to S and $\phi(1_R) \neq 1_S$, prove that $\phi(1_R)$ is is a zero-divisor in S.

10. Show that $\mathbb{Z}[x]/\langle x^2 + 1\rangle$ is ring isomorphic to $\mathbb{Z}[i]$.