Exam 1 is an in class exam to be given on Tuesday May 7th.

- Exam 1 covers Chapter $8, 10, 11, 12, 13$.
- You may have one side of one sheet of notes. You may have no more than 2 worked out problems or theorem proofs on your note card. You will turn in your notes with your exam.
- Suggestions for study:
  - Review the theorems and proofs from the class and book. Work out the proofs on your own, then check with the book or notes.
  - Redo (not just look at) assigned homework problems.
  - Do additional problems from the text.
  - Work out the practice problems below. Make up a new sheet of practice problems and trade with a friend.
  - Make a sheet of notes.
  - Practice problems in a timed environment. Redo problems until you can do them quickly without looking at notes. This better simulates the exam environment.
- **Disclaimer:** The set of problems below is not meant to be an exhaustive list of the type of problems that may be on the exam, it is simply for your practice.

SET 1

1. How many elements of order 2 are there in $D_6 \oplus D_4$?

2. What is the largest order for *any* element in $Z_{15} \oplus Z_{18} \oplus Z_8$?

3. Find two subgroups of $Z_{20} \oplus Z_4 \oplus Z_9$ that are isomorphic to Z_{30}.

4. How many *subgroups* of order 6 does $Z_{12} \oplus Z_8$ have?

SET 2

5. If ϕ is a homomorphism from Z_{40} onto a group of order 8, determine the kernel of ϕ.

6. If ϕ is a homomorphism from Z_{40} to a group of order 6, determine the possibilities for the kernel of ϕ.

7. Determine all homomorphisms from Z_{12} to Z_{20}.

8. Suppose that ϕ is a homomorphism from a finite group G onto Z_{10}. Prove that G has normal subgroups of indexes 2 and 5.

9. Let $\phi : G \to \overline{G}$ be a group homomorphism. If $\overline{K}$ is a normal subgroup of $\overline{G}$ prove that $\phi^{-1}(\overline{K})$ is a normal subgroup of G.

SET 3

10. Write down all abelian groups of order 36. Which ones are cyclic?

11. Prove or disprove: $Z_{36} \oplus Z_{30} \approx Z_{60} \oplus Z_{18}$.

12. $H = \{1, 17, 41, 49, 73, 89, 97, 113\}$ is a group under multiplication modulo 120. Write H as an external direct product of groups.

13. Suppose that R is a ring with more than one element and with unity, 1. Prove that $0 \neq 1$.

SET 4

14. List all the zero divisors of $Z_6 \oplus Z_3 \oplus Z$. List all the units.

15. What is the characteristic of a field with 625 elements?

16. Suppose that there is an r in a ring such that $r^2 = r$. Prove that r or $1-r$ is NOT a unit.

17. Recall that an idempotent is an element a of a ring such that $a^2 = a$. In a commutative ring of characteristic 2, prove that the idempotents form a subring.

SET 5

18. Suppose that a and b are elements of an integral domain. If $a^7 = b^7$ and $a^2 = b^2$ prove that $a = b$.

19. What is the characteristic of the ring $Z_4 \oplus Z_{18}$?

20. Show that any finite field must have order p^n where p is a prime. Hint: Use facts about abelian groups.

21. Can a finite field have characteristic 0? Explain.