

MTH 345 Final Exam In-Class Review Problems

1. Prove or Disprove: $\mathbb{R}[x]/\langle x^4 - 4 \rangle$ is a field.
2. (a) Prove $2\mathbb{Z} \cup 3\mathbb{Z}$ is **not** a subring of \mathbb{Z} .

(b) Prove $2\mathbb{Z} \cap 3\mathbb{Z}$ **is** a subring of \mathbb{Z} .
3. What is the characteristic of the ring \mathbb{Z}_{323} ?
4. (a) Write down all the elements of \mathbb{Z}_8 .
(b) Write down all the elements of $\mathbb{Z}_4 \oplus \mathbb{Z}_2$.
(c) Write down all the elements of $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$.
(d) Write down all the elements of $\mathbb{Z}_3[i]$.
(e) It is a fact that the non-zero elements of $\mathbb{Z}_3[i]$ form a group under multiplication.
 - i. Find the multiplicative inverse of $1 + i$.
 - ii. Find the (multiplicative) order of $1 + i$.
- (f) The non-zero elements of $\mathbb{Z}_3[i]$ form an abelian group of order 8 under **multiplication**. Is this group isomorphic to \mathbb{Z}_8 , $\mathbb{Z}_4 \oplus \mathbb{Z}_2$, or $\mathbb{Z}_2 \oplus \mathbb{Z}_2 \oplus \mathbb{Z}_2$? (Note these are groups under component-wise **addition**.) Justify your answer.
5. Prove that $f(x) = \frac{4}{13}x^3 + \frac{5}{7}x - 1$ is irreducible over $\mathbb{Q}[x]$.
6. Let F be a field. Prove or disprove that F can have 7 elements of order 5. (To prove, it is enough to give an example.)
7. Let $\phi : R \rightarrow S$ be a ring homomorphism.
 - (a) Prove that if A is an ideal of R and ϕ is onto, then $\phi(A)$ is an ideal of S .
 - (b) Does the result still hold if ϕ is not onto? Explain.

MTH 345 Final Exam Comments

- The final exam is an in-class exam given on Wednesday, June 13, 2012 from 12 : 00 – 1 : 50pm in MNB 104.
- You may use one page of notes (regular size paper) both sides with at most 3 worked out examples or proofs. You will turn in your notes with your exam.
- The exam is cumulative and covers the topics we covered in Chapters 10, 12 – 17.
- Suggestions for study:
 - Write out summaries of each chapter and include important ideas, examples and theorems.
 - Review the theorems and proofs from the class and book. Work out the proofs on your own, then check with the book or notes.
 - Redo (not just look at) assigned homework problems.
 - Do additional problems from the text:
 - * Write out the problems from various sections on slips of paper and then put your book away. Mix up the sheets and try to do the problem without looking at the book (you might want to write the section and problem number on the back of the paper so you can check your answer).
 - * Examples that are worked out in the section are good ones to put down as you can go back and look at how the book does the problem.
 - * Also note there are supplementary exercises for the chapters as well.
 - Make up a set of T/F problems and trade them with your friends.
 - Work out the practice problems given in class.
 - Make a sheet of notes.