• **Definition of Group** Let $G$ be a set and $\circ$ a binary operation on $G$ that assigns to each ordered pair $(a, b)$ of elements of $G$ an element in $G$ denoted by $a \circ b$. We say that $G$ is a **GROUP** under the operation $\circ$ if the following three properties are satisfied:

1. **Associative.** The operation is associative on the set $G$: $\forall a, b, c \in G, (a \circ b) \circ c = a \circ (b \circ c)$
2. **Identity.** There is an element $e \in G$ (called the identity element) such that $a \circ e = e \circ a = a$ for all $a \in G$.
3. **Inverses.** For each $a \in G$ there is an element $b \in G$ such that $a \circ b = b \circ a = e$ ($b$ is called the inverse of $a$).

- Note that “hidden” in this definition is that the $\circ$ is CLOSED on $G$. Don’t forget to check that property.
- We say the group is **Abelian** if it is commutative: $a \circ b = b \circ a \ \forall a, b \in G$.

**Directions:** Prove each of the following.

1. **Theorem 2.1 Uniqueness of the Identity** In a group $G$, there is only one identity element.

2. **Theorem 2.2 Cancellation** In a group $G$, the right and left cancellation laws hold; that is, $ba = ca$ implies $b = c$ and $ab = ac$ implies $b = c$. 
3. **Theorem 2.3 Uniqueness of Inverses** For each element $a$ in a group $G$, there is a unique element $b$ in $G$ such that $ab = ba = e$. (Unless otherwise noted, we will always assume $e$ is the identity element of the group.)

4. **Theorem 2.4 Socks-Shoes Property** For group elements $a$ and $b$, $(ab)^{-1} = b^{-1}a^{-1}$. 