Exam 2 is an in class exam to be given on Monday, March 11th.

- Exam 2 covers Chapters 5 – 7 and the last part of Chapter 4 (Thm. 4.4 and its Corollary).
- You may have one sheet of notes, one side only (regular size paper). You may have no more than 3 worked out problems or theorem proofs on your note sheet. You will turn in your sheet of notes with your exam.
- If you need it, I will provide a copy of the back cover of your boook (the Cayley tables for D_4 and D_3) or of the elements of A_4.
- Suggestions for study:
  - Review the theorems and proofs from the class and book. Work out the proofs on your own, then check with the book or notes.
  - Redo (not just look at) assigned homework problems.
  - Do additional problems from the text.
  - Work out the practice problems below. Make up a new sheet of practice problems and trade with a friend.
  - Make a sheet of notes.
  - Practice problems in a timed environment. Redo problems until you can do them quickly without looking at notes. This better simulates the exam environment.
- **Disclaimer:** The set of problems below is not meant to be an exhaustive list of the type of problems that may be on the exam, it is simply for your practice.

1. (a) TRUE FALSE S_n is non-Abelian for all $n \geq 3$.

   (b) TRUE FALSE If a is a permutation that is an $m-$cycle and b is a permutation that is an $n-$cycle, then $|ab| = lcm(m, n)$.

   (c) TRUE FALSE If a group has an element of order 10, then the number of elements of order 10 is divisible by 4.

   (d) TRUE FALSE A $1-1$ mapping from a set to itself is onto.

   (e) TRUE FALSE If a finite group has order n then the group contains a subgroup of order d for every divisor d of n.

   (f) TRUE FALSE If H is a subgroup of G and a and b belong to G, then aH and Hb are either identical or disjoint.

   (g) TRUE FALSE If H is a subgroup of a finite group G, then the number of distinct left cosets of H in G divides $|G|$.

   (h) TRUE FALSE A group can be isomorphic to a proper subgroup of itself.

   (i) TRUE FALSE Two groups isomorphic to the same group are isomorphic to each other.

2. Give an example of a group that has subgroups of orders $1, 2, 3, 4, 5$, and 6 but does not have a subgroup of order 7 or 8.

3. Find the order of the permutation $\alpha = (124)(2345)$. Is α even or odd? What is α^{16} (don't compute it out, use some theorems)

4. In the group S_n, let $\alpha = (12)(123)(1234)(12345)...(123...n)$. If $n = 99$, determine whether α is even or odd.

5. Suppose that ϕ is an automorphism of Z_9 (isomorphism of Z_9 to itself) and $\phi(4) = 1$. Determine a formula for ϕ.

6. Find all the left cosets of $\{1, 11\}$ in $U(20)$.

7. Given that $|a| = 20$, find all left cosets of $\langle a^{12} \rangle$ in $\langle a \rangle$.

8. Let p be a prime and let n be a positive integer. How many subgroups does Z_{p^n} have (including the trivial subgroup and the group itself)?