MTH 344 Assumptions

Retain this sheet for reference.

Notation $Z$ is the set of all integers. $Q$ is the set of all rational numbers. $R$ is the set of all real numbers. $C$ is the set of all complex numbers.

Putting a $*$ after any set of numbers means the set of all nonzero elements of that set. For example, $R^*$ is the set of all nonzero real numbers.

Putting a $+$ after any of real numbers means the set of all positive elements of that set. For example, $R^+$ is the set of all positive real numbers. Recall that the number 0 is neither positive nor negative.

Assumptions Here are the assumptions we take as our starting place in the course:

1. $+$, $-$ and $\times$ are defined and well-defined on any subset of $Z$, $Q$, $R$ and $C$.

2. $+$, $-$ and $\times$ are closed on $Z$, $Q$, $R$ and $C$ (Of course, $+$, $-$ and $\times$ are not necessarily closed on a subset of those sets. It depends on the subset.)

3. $\div$ is defined and well-defined on any subset of $Z^*$, $Q^*$, $R^*$ and $C^*$.

4. $\times$ and $\div$ are closed on $Q^*$, $R^*$ and $C^*$ ($\times$ and $\div$ are not necessarily closed on a subset of those sets. It depends on the subset.)

5. $+$, $\times$ and $\div$ are closed on $Q^+$ and $R^+$ ($+$, $\times$ and $\div$ are not necessarily closed on a subsets of those sets. It depends on the subset.)

6. $+$ and $\times$ are associative and commutative on any subset of $Z$, $Q$, $R$ and $C$.

7. The distributive property holds on any subset of $Z$, $Q$, $R$ and $C$.

Additional Assumptions

8. $Z$, $Q$, $R$ and $C$ are Abelian groups under $+$.

9. $Q^*$, $R^*$ and $C^*$ are Abelian groups under $\times$.

10. $+_n$ and $\times_n$ are associative binary operations on $Z_n$.

11. $Z_n$ is an Abelian group under $+_n$.

12. If $a, b \in C$ and $ab = 0$, then either $a = 0$ or $b = 0$. (Note that this assumption applies automatically to any subset of $C$, including $Q$ and $R$ and $Z$.)

13. Arithmetic facts (like $2 + 3 = 5$ and–for now–rules of signs, for example)

14. Basic facts about the familiar functions of college algebra and trig, ln, sin, etc. including the assumption that they are functions.

15. The quadratic formula.

16. $\sqrt{2}$ is not an element of $Q$. In general, $\sqrt{p}$ is not an element of $Q$ when $p$ is a prime number.

17. For every $a \in C$, $\sqrt{a} \in C$. For every $a \in R^+$, $\sqrt{a} \in R^+$.

\footnote{Shamelessly stolen from Mike Ward}