MTH 344 Exam 2 Review

Exam 2 is an in class exam to be given on Tuesday, Nov. 22nd.

- Exam 2 covers Chapters $5-7$.
- You may have one sheet of notes, one side only (regular size paper). You may have no more than 3 worked out problems or theorem proofs on your note sheet. You will turn in your sheet of notes with your exam.
- If you need it, I will provide a copy of the back cover of your boook (the Cayley tables for D_4 and D_3) or of the elements of A_4.
- I will provide a list of the theorems and corollaries we covered from chapters $4-7$.
- Suggestions for study:
  - Review the theorems and proofs from the class and book. Work out the proofs on your own, then check with the book or notes.
  - Redo (not just look at) assigned homework problems.
  - Redo problems done in class or on a quiz.
  - Do additional problems from the text.
  - Work out the practice problems below. Make up a new sheet of practice problems and trade with a friend.
  - Make a sheet of notes.
  - Practice problems in a timed environment. Redo problems until you can do them quickly without looking at notes. This better simulates the exam environment.
- **Disclaimer:** The set of problems below is not meant to be an exhaustive list of the type of problems that may be on the exam, it is simply for your practice.

1. (a) TRUE FALSE If a is a permutation that is an $m-$cycle and b is a permutation that is an $n-$cycle, then $|ab| = lcm(m, n)$.
   (b) TRUE FALSE A $1-1$ mapping from a set to itself is onto.
   (c) TRUE FALSE If a finite group has order n then the group contains a subgroup of order d for every divisor d of n.
   (d) TRUE FALSE A group can be isomorphic to a proper subgroup of itself.
   (e) TRUE FALSE Two groups isomorphic to the same group are isomorphic to each other.

2. Let $\beta = (12346)(1345)(2643)$
   (a) Write β in array notation.
   (b) What is the order of β?
   (c) Is β an even or odd permutation?
   (d) What is β^{-1}?

(e) What is β^{101}?

(f) What is the order of $\alpha = (134)(5678)$?

3. How many elements are there in S_4?

4. How many elements of order 2 are there in S_4? Write them all down.

5. Suppose $\phi : Z_{12} \to Z_{12}$ with $\phi(7) = 3$. Prove or disprove that ϕ is an isomorphism.

6. Prove or disprove that $D_4 \cong S_4$.

7. Let G be a finite group and $a \in G$ a fixed element of G. Prove that the mapping $\phi : G \to G$ given by $\phi(x) = axa^{-1}$ is an automorphism of G.

8. Suppose $\phi : U(7) \to U(7)$ is an automorphism such that $\phi(3) = 5$. Write down the values of $\phi(x)$ for all $x \in U(7)$.

9. Suppose $A = \langle a \rangle$ and $B = \langle b \rangle$ are finite cyclic groups with $|a| = |b| = n$, then prove that $\phi : A \to B$ such that $\phi(a^k) = b^k$, for $k = 0, 1, ..n - 1$ is an isomorphism.

10. (a) Prove that the mapping $\phi : \mathbf{Z}_n \to \mathbf{Z}_n$ such that $\phi(x) = kx$ is an automorphism if and only if $k \in U(n)$.

    (b) Use the above to write down all automorphisms of $\mathbf{Z}_{12}$.

11. Suppose $G = \langle a \rangle$ is a cyclic group of order 6. If ϕ is an automorphism of G and $\phi(a^5) \neq a^5$,then what must $\phi(a^5)$ be?

12. Prove that if $\phi : A \to B$ is an isomorphism from A to B and $\alpha : B \to C$ is an isomorphism from B to C, then $\alpha \circ \phi : A \to C$ is an isomorphism from A to C. Hint: You may use Theorem 0.8.

13. Find or prove there does not exist an element of order 6 in A_8, the group of even permutations on $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

14. In the group S_n, let $\alpha = (12)(123)(1234)(12345)\cdots(123...n)$. If $n = 99$, determine whether α is even or odd.

15. Write down all of the left cosets of $\langle 4 \rangle$ in $\mathbf{Z}_{16}$.

16. Suppose G is a group of order 60 and H is a subgroup of order 10. Prove that for every $a \in G$, there is an integer k with $1 \leq k \leq 6$ such that $a^k \in H$. (Hint use Cor. 1 to Thm. 7.1 and the Ch. 7 Lemma).

17. Use Corollary 4 to Thm. 7.1 to find $6^{25} \pmod{15}$. Be sure you understand how you are using Corollary 3.

18. Suppose $G = \langle a \rangle$ is a cyclic group of order 20. Let $H = \langle a^5 \rangle$. Then is $a^2H = a^8H$? Justify your answer using the chapter 7 Lemma.