

Math 251: Review Problems for Exam 2

Exam 2 is an in class exam given Thursday, November 20th, and covers Sections 2.6-2.9 and 3.1 to 3.6. You may use both sides of one 3x5 notecard.

1. Find the derivatives of the following functions. Combine like terms when simplifying, use positive exponents.

(a) $\sin(3x^2)\cos(x)$

(b) $2^x \tan(x)$

(c) $e^{x^2} + \frac{2}{\sqrt{x}} + \frac{1}{x}$

(d) $\frac{x^2 + 1}{e^{2x}}$

(e) $\frac{\cos(x)}{3 + 7x}$

(f) $(3x^2 + 2x)^9(5x + 6)^4$

(g) $\sqrt[3]{x^2 + 4x}$

(h) $\sin(\sqrt{x})$

(i) $\frac{3}{(x^3 + 2x^2 + 1)^{10}}$

(j) $\csc(x)$ (derive the formula)

(k) $x^2 e^x \cos(x)$

(l) $x^2 y + 3y^2 = 4xy$ (use implicit differentiation)

(m) $(y^2 + x^2) = y$ (use implicit differentiation)

2. Find the equation of the line tangent to the curve $y = e^x(x^2 - 3)$ at the point $(0, 3)$.
3. Find the equation of the line tangent to the curve $y + y^2 x - 6x^2 = 0$ at the point $(1, 2)$. Use implicit differentiation.
4. A table of values for f, g, f' , and g' is given below:

x	f(x)	g(x)	f'(x)	g'(x)
1	1	3	2	6
2	3	4	3	-2
3	4	3	1	3
4	2	1	-1	-2

(a) If $h(x) = f(x)g(x)$, find $h'(1)$

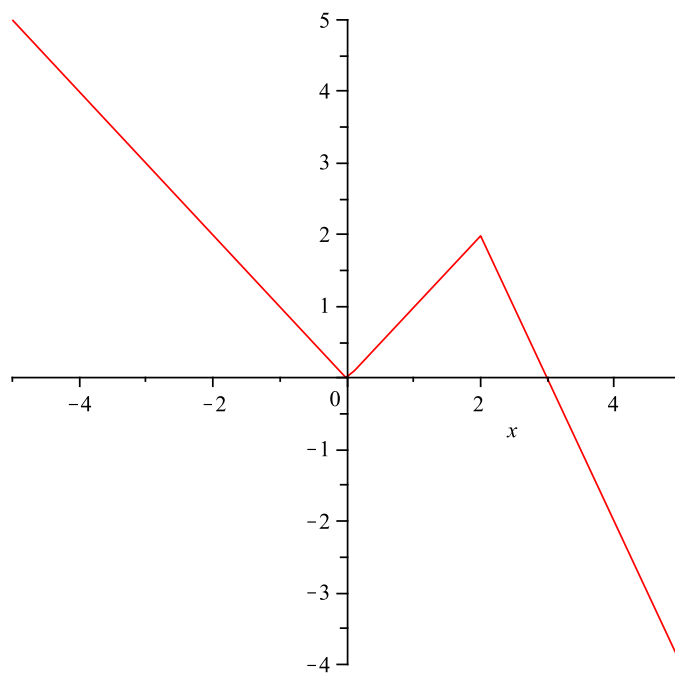
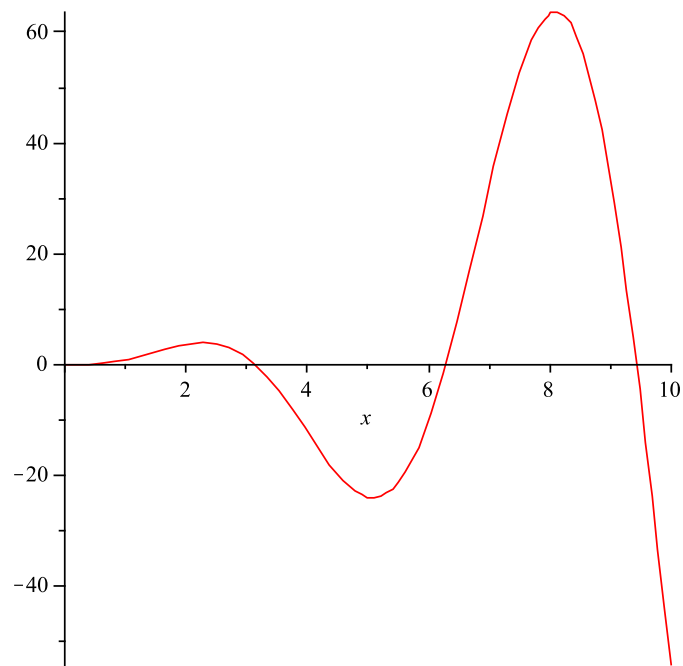
(b) If $h(x) = \frac{f(x)}{g(x)}$, find $h'(2)$

(c) If $h(x) = f(g(x))$, find $h'(1)$

(d) If $h(x) = 3f(g(2x))$, find $h'(1)$

(e) If $h(x) = g(f(x^2))$ find $h'(2)$

5. Below are graphs of various functions, $f(x)$. For each, sketch a graph of the derivative, $f'(x)$.



6. The position of a particle is given by the equation

$$s = f(t) = t^3 - 6t^2 + 9t$$

where t is measured in seconds and s in meters.

- Find the velocity at time t .
 - What is the velocity after 2s? After 4s?
 - When is the particle at rest?
 - When is the particle moving forward (that is in the positive direction)?
 - Draw a diagram to represent the motion of the particle.
 - Find the total distance traveled by the particle during the first five seconds.
 - Find the acceleration at time t and after 4s.
 - Graph the position, velocity and acceleration functions for $0 \leq t \leq 5$.
 - When is the particle speeding up? When is it slowing down?
7. The following formula gives the population of mice in thousands on a particular island at time t where t is measured in years:

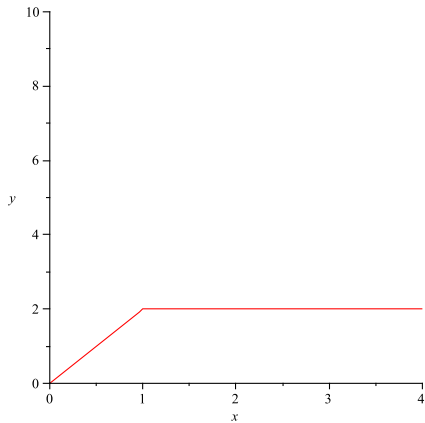
$$P(t) = -t^2 + 12t + 2$$

- When is the population increasing? Explain.
 - Find $P'(1)$ and $P'(7)$. Explain what your answers mean in terms of population and growth.
 - Does the population continue to grow or eventually die out? Explain.
8. Sketch a graph of a function, $f(x)$, with the following properties:
- $f(0) = 0$
 - $f'(x) < 0$ on the interval $(-\infty, 0)$
 - $f'(x) > 0$ on the interval $(0, \infty)$
 - $f'(x)$ is NOT differentiable at $x = 0$

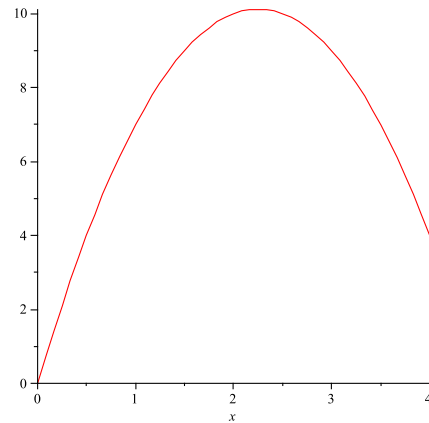
9. Sketch a graph of a function, $f(x)$, with the following properties:

- $f''(x) > 0$ on the interval $(0, 1)$
- $f''(x) < 0$ on the interval $(1, \infty)$
- $f'(x) > 0$ on the interval $(-\infty, 0)$

10. The following are the graphs of the *velocity* function of two particles at time t seconds, $0 \leq t \leq 4$.



Particle A



Particle B

- When is particle (A) speeding up? Explain.
- Assuming the particles start at the same point, which particle has gone farther after 4 seconds? Explain.

For more review problems see the appropriate section of your text or

- Chapter 2 Review, page 175-178, Problems 32-34,38,41-43
- Chapter 3 Review, page 255-257 (Note: Sections 2.7 and 2.8 not on exam)