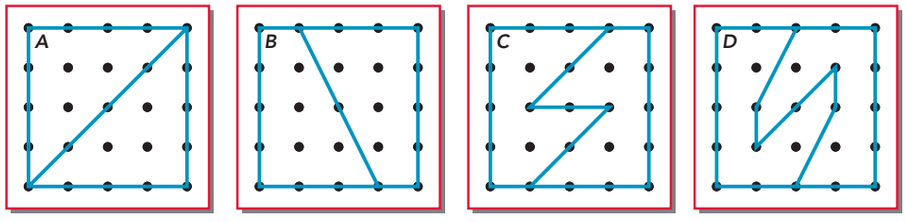
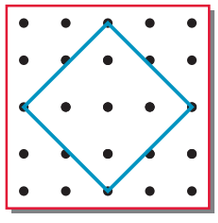
Follow-Up Questions and Activities 9.1

1. *School Classroom:* While working on an activity to divide geoboard squares into two congruent halves, several students could see that halves as in figures A and B were congruent but could not see that the parts of figures C and D were also congruent halves. Explain how you can help students determine when two plane figures are congruent so they can apply this knowledge to geoboard figures as well as other plane figures.



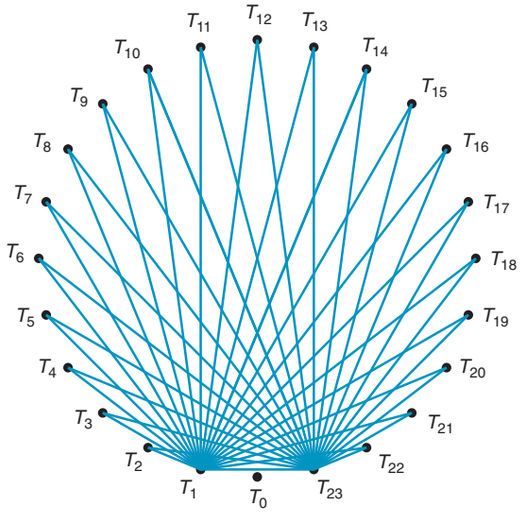
2. *School Classroom:* Several of your students insist that the following is a nonsquare rhombus because the sides are at a diagonal. How can you help your students resolve this issue?



3. *School Classroom:* The second grade **Elementary School Text** page at the beginning of this section asks students to draw geometric shapes. Suppose you had students in your class who could recognize triangular, square, and rectangular shapes but were not able to dissect the **Elementary School Text** page figures into different shapes as requested. Describe at least two different ways you could help these students.

4. *Math Concepts:* Each triangle in the following family of triangles formed on a circular geoboard has base T_1T_{23}. Downloadable Circular Geoboards and printable Virtual Geoboards are available at the companion website.

   Determine which triangles are congruent and which triangles are acute, obtuse, scalene, isosceles, equilateral, or right; identify each triangle by its vertex angle (T_2 through T_{22}) and list the triangles in each category. Sketch or print the triangles you have formed and explain why each triangle belongs in the category you have put it in.



5. *Math Concepts:* Which regular polygons (all sides congruent and all angles congruent) can you form on a 24-pin circular geoboard? Illustrate each regular polygon on a circular geoboard and explain how you know it is a regular polygon. Circular Geoboards are available for download at the companion website.

6. *Math Concepts:* Downloadable Rectangular Geoboards and printable Virtual Geoboards are available at the companion website.
   a. Form polygons with 13, 14, 15, and 16 sides whose vertices are pins of a 5 pin by 5 pin geoboard. Sketch or print your work.
   b. Classify the polygons in part a as convex or concave.
   c. What is the polygon with the most number of sides you can form with vertices on the pins of a 5 pin by 5 pin geoboard? Is the polygon convex or concave? Sketch or print your work.

7. *NCTM Standards:* Go to http://illuminations.nctm.org/ and under "Lessons" select grade levels 3–5 and the **Geometry Standard.** Choose a lesson that involves polygons.
   a. State the title of the lesson and briefly summarize the lesson.
   b. Referring to the **Standards Summary** in the back pages of this book as necessary, list the *Geometry Standard Expectations* that the lesson addresses and explain how the lesson addresses these *Expectations*.

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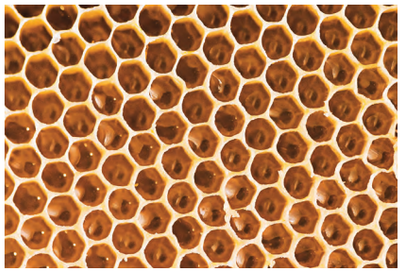
Follow-Up Questions and Activities 9.2

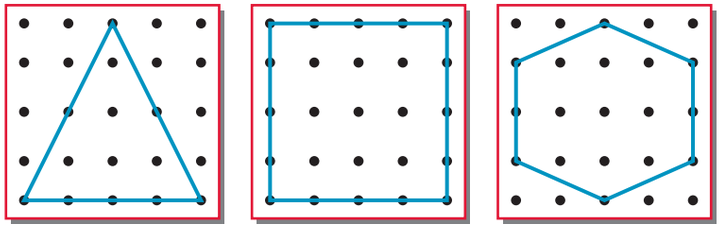
1. *School Classroom:* Bees form tessellations when they create their honeycombs. How might you use this idea to design an activity for your classroom? Explain your activity ideas and your thinking.



(a)

(b)

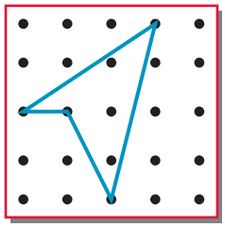
2. *School Classroom:* One of your students claims that she has formed regular polygons of 3, 4, and 6 sides on her rectangular geoboard and shows you the following figures. Explain how you would help her determine which of these figures are regular and which are not.



3. *School Classroom:* Do an Internet search and find a website that features a game or math applet focused on tessellations for kids. Print a one-page view of the game or applet, describe the game or applet, and discuss how you would use such an activity in your own class. Give the URL of the game or math applet you found.

4. *Math Concepts:* On Dot Paper or Grid Paper draw a seven-sided polygon that tessellates and show the tessellation. Dot Paper and Grid Paper are available for download at the companion website.

5. *Math Concepts:* Copy this concave quadrilateral in the center of a page of dot paper. Dot Paper is available for download at the companion website.



   a. Form a tessellation with this figure. Draw enough of the tessellation so it is obvious that the tessellation can be continued in all directions. Dot Paper is available for download at the companion website.

   b. What observations can you make about the angles around the points where the vertices of the quadrilaterals meet?

6. *Math Concepts:* Use Polygons for Tessellations and create a semiregular tessellation different from the tessellation you produced for activity 5a. Sketch or print your work. Printable Virtual Polygons for Tessellations are available at the companion website.

7. *NCTM Standards:* Read over the **Standards Summary** in the back pages of this book. Tessellations are not mentioned. What *Content Standards* and *Expectations* does the study of these topics address? List the *Standards* and *Expectations* and explain your thinking.

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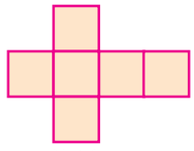
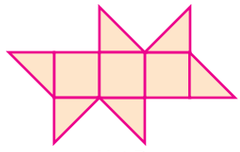
Follow-Up Questions and Activities 9.3

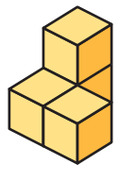
1. *School Classroom:* Suppose that your students are using sticks and gumdrops to create models of the five Platonic solids. How will you help them decide how many sticks and how many gumdrops they need and how to assemble their solids?

2. *School Classroom:* When asked to make a net for a cube, a drawing like net A is the usual response. Design an activity for middle school students that will lead them to draw nets like net B that are formed without using all squares. Describe your activity in such a way that another person could follow your directions. Submit a few designs for a net by sketching them on 2-Centimeter Grid Paper. 2-Centimeter Grid Paper is available for download at the companion website.


Net A


Net B

3. *Math Concepts:* Design a two-dimensional net for the following figure that is composed of four cubes. Describe your procedure for constructing this net and submit a copy of the net with your response.



4. *Math Concepts:* Open the **Math Laboratory Investigation 9.3: Read Me—Pyramid Patterns Instructions** from the companion website and investigate the pyramid patterns described in question 1 of the *Starting Points for Investigations 9.3.* Show your procedures and explain your thinking.

5. *Math Concepts:* Check your conjecture for Euler's formula (activity 3a) on the two-polygon Archimedean solids in activity 5 (parts b, d, e, f, g, h, i, j, and l) by listing and comparing the number of edges, vertices, and faces. You may wish to do an Internet search on a site such as Math World (mathworld.wolfram.com) to find movable three-dimensional views of the solids pictured here.

6. *NCTM Standards:* Read over the **Geometry Standards** in the back pages of this book. Pick one *Expectation,* from each grade level that the activities in this section address. State the *Expectations* and the *Standards* they are under. Explain which activities address these *Expectations* and how they do so.

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Follow-Up Questions and Activities 9.4

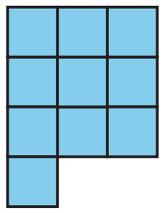
1. *School Classroom:* One of your students claims that any line drawn through the center of a square is a line of symmetry for the square. Describe what you believe this student was thinking and how you would help her determine the lines of symmetry of a square without actually showing her these lines.

2. *School Classroom:* Rydell has found a wonderful pattern. Squares have four lines of symmetry and four rotational symmetries, nonsquare rectangles have two lines of symmetry and two rotational symmetries. He is sure this pattern (the number of lines of symmetry equals the number of rotational symmetries) holds for all quadrilaterals. Is he correct and if not, how can you help him resolve this issue?

3. *School Classroom:* Determine and describe the rotational and line symmetries for each of the four figures in *Try It* at the bottom of the **Elementary School Text** page at the beginning of this section.

4. *Math Concepts:* For each of the following use any combination of Pattern Blocks (cardstock or virtual) to form the figure with the stated properties; but do not duplicate any figures formed in this activity set. Sketch or print your work. Printable Virtual Pattern Blocks are available at the companion website.
   a. Figure a: One line of symmetry and no rotational symmetries (other than 360°).
   b. Figure b: Six rotational symmetries but no lines of symmetry
   c. Figure c: Three rotational symmetries and three lines of symmetry
   d. Figure d: No rotational symmetries (other than 360°) and no lines of symmetry.

5. *Math Concepts:* How many symmetrical shapes can be made by joining one more square to the ten square shape here? Use color tiles from the Manipulative Kit to find the shapes and record your answers on centimeter grid paper. Describe the type of symmetry. Centimeter Grid Paper is available for download at the companion website.



6. *Math Concepts:* Open the **Math Laboratory Investigation 9.4: Read Me—Mirror Cards Instructions** from the companion website and investigate the mirror patterns described in 1, 2, and 3 of the *Starting Points for Investigations 9.4.* Show your procedures and explain your thinking. A small handheld mirror will be helpful.

Follow-Up Questions and Activities 10.1

1. *School Classroom:* In a half page or more, make a case for one of the following approaches to teaching the metric system.
   a. The metric system should be taught from the point of view that it is secondary to the standard English system and learning how to convert between the two systems is the primary goal.
   b. The metric system should be taught so students can operate in the metric system with the facility we expect them to have with the standard English system.

2. *School Classroom:* While doing a measurement activity, one of your students asks you what it means to record a measurement to the nearest centimeter. Explain how you would help him understand what it means to measure "to the nearest centimeter."

3. *School Classroom:* Many measuring activities using millimeters and centimeters can be done *inside* a school classroom. What common objects can students estimate or measure *outside* a school classroom to develop a facility with measuring with meters? Read the **Elementary School Text** page at the beginning of this section and then describe two estimation and measuring activities using meters that students can do outside of their school classroom.

4. *Math Concepts:* In the metric system of measures, "meters" are used instead of "yards" from the English system. What used to be the 100-yard dash in track has now become the 100-meter dash (even though the distances are different).
   a. What lengths in the metric system do you think are used instead of these common units of length in the English system: quarter of an inch, inch, foot, and mile?
   b. Give examples to explain the advantage of the metric system over the English system with respect to the numerical relationship between the common units of length within each system.

5. *Math Concepts:* The Celsius scale is used for temperature in the metric system. Freezing is 0C and boiling is 100C on the Celsius scale. The corresponding temperatures on the Fahrenheit scale are 32° and 212° respectively. Viewing the two temperature scales side-by-side as below, and using arithmetic but no formulas, explain how you can determine the equivalent Fahrenheit temperature for 20C.

6. *NCTM Standards:* Go to http://illuminations.nctm.org/ and under "Lessons" select grade levels 6–8 and the **Measurement Standard.** Choose a lesson involving measurement.
   a. State the title of the lesson and briefly summarize the lesson.
   b. Referring to the **Standards Summary** in the back pages of this book as necessary, list the *Measurement Standard Expectations* that the lesson addresses and explain how the lesson addresses these *Expectations*.

7. *NCTM Standards:* Using the **Standards Summary** at the back of this book, describe what metric measuring experiences are being suggested at each level: Pre-K–2, Grades 3–5, and Grades 6–8.

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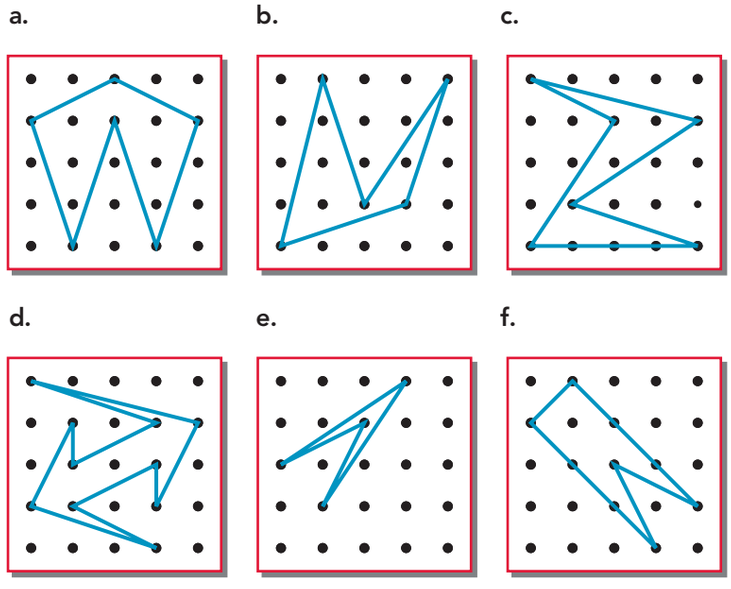
Follow-Up Questions and Activities **10.2**

Downloadable Dot Paper, Rectangular Geoboards and printable Virtual Rectangular Geoboards are available at the companion website.

1. *School Classroom:* One of your students says that she can find the area of any geoboard figure that has no pins inside of it by counting the number of pins on the boundary of the figure, subtracting 2, and then dividing that number by 2. Form and record several figures that satisfy her conditions and determine their areas to test her conjecture. What would you say to this student?

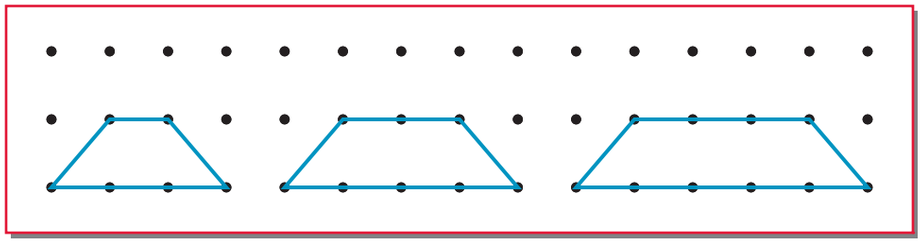
2. *School Classroom:* Design a geoboard activity connecting the exploration of perimeter and area in parallelograms that you believe is appropriate for an elementary school student and write a few questions that you can ask about your activity. Try this activity on an elementary school age child of your choice. Describe your activity and record your questions and the student's responses.

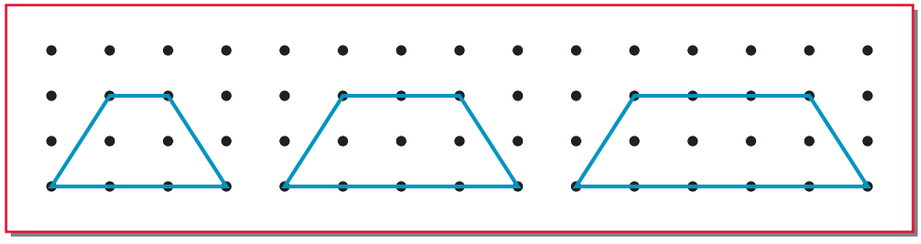
3. *Math Concepts:* Determine the areas of the following figures using any technique or combination of techniques from the activities in this section. Notice that the hexagon in part a has a vertical line of symmetry, so you can determine the area of half the hexagonal region and then double the result to obtain the area of the entire hexagonal region.



4. *Math Concepts:*

a. Use geoboard techniques to determine the areas of these six trapezoids without using any known formulas (see Activity Set 9.1 for the definition of trapezoid).





b. Make a conjecture about the area of trapezoids with upper bases of equal length and lower bases of equal length but with different heights.

c. Sketch three additional noncongruent trapezoids and determine their areas to test your conjecture.

5. *Math Concepts:* On a 5 by 5 geoboard:

a. Determine a triangle of *least* area that can be formed with vertices on pins. Repeat for the triangle of greatest area. Record your triangles and justify your answers.

b. Is it possible to form a triangle with area 1 square unit and a triangle with area 8 square units on a geoboard? Is it possible to form triangles with area 2, 3, 4, 5, 6, or 7 square units on a geoboard? Record your triangles and justify your answers.

6. *NCTM Standards:* Using the **Standards Summary** at the back of this book, list the *Expectations* that suggest using activities with non-standard units of measurements (such as the width of your hand span) for Pre-K–2. Determine if these ideas extend to Grades 3–5 and Grades 6–8 and describe your results.

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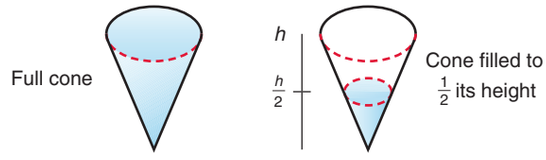
Follow-Up Questions and Activities 10.3

1. *School Classroom:* Katrina is convinced that the two cylinders made by rolling a standard piece of paper left-to-right or top-to-bottom, must have the same volume. Describe how you can help Katrina explore this concept without using any formulas.

2. *School Classroom:* Make paper models to illustrate the following relationships and explain how you would illustrate the relationships in classroom experiences with maximum student involvement.
   a. The volume of a square pyramid is one-third the volume of a box with the same base and the same height as the pyramid.
   b. The volume of a right circular cone is one-third the volume of a cylinder with the same base and the same height as the cone.
   c. The volume of an inverted right circular cone has eight times the volume as an identical inverted cone that is filled to one-half its height.



3. *School Classroom:* The surface area activity described on the **Elementary School Text** page at the beginning of this section assumes the students know certain formulas for area. Describe how you can restructure this activity so that students can obtain an approximation of the surface area without knowing any area formulas.

4. *Math Concepts:* If you have a cubical box that measures 3 inches on each edge and want to make a box with three times the volume, how long must each edge of the larger box be (to the nearest tenth of an inch)? Explain your thinking.

5. *Math Concepts:* Imagine that you have a cube of modeling clay that measures 4 inches on each edge.
   a. What is the surface area of that cube of clay?
   b. If you formed the same piece of clay (same volume) into a sphere, predict which clay object would have more surface area, the cube or the sphere. Then compute the surface area of the sphere to test your prediction (surface area of a sphere is $4\pi r^2$). Explain in detail how you determined the surface area of the clay sphere.

6. *Math Concepts:* Go to the companion website and open the Chapter 10 Applet: Filling 3D Shapes. Compare the volumes of the various shapes and record your observations.

7. The **Standards Summary** at the back of this book mentions *Volume* at every grade level. Give three simple examples of grade level appropriate activities exploring volume, one each for Pre-K–2, grades 3–5, and grades 6–8. Explain how these activities address *Expectations* in the **Geometry Standards.** State the *Expectations* with your explanation.

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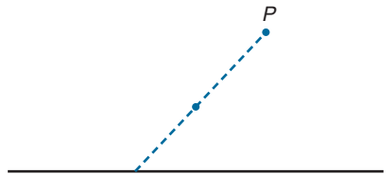
Follow-Up Questions and Activities 11.1

1. *School Classroom:* Cresedeo is having trouble sorting out the following four geometric terms: circle, diameter, chord, and arc. Describe several hands-on ways you might use to help Cresedeo and other students make sense of these terms.

2. *School Classroom:* One way to introduce students to definitions involving sets of points in the plane is to ask them to place or locate small objects (beans, rice, counters, etc.) that satisfy certain conditions. For example, placing a bean on a table and having your students place many pieces of rice all an equal distance from the bean leads to the definition of a circle. Create two activities concerning the location of sets of points that are appropriate for the elementary school classroom. Describe your activities in detail and accompany them with diagrams.

3. *Math Concepts:* Analyze the line design at the beginning of the **Just for Fun Activity** in this section. Write a set of directions that will enable your reader to construct this figure. Using your directions, construct the figure. If you refine your directions while drawing, explain the refinement.

4. *Math Concepts:* Suppose a point P is placed above a line and many line segments are drawn from P to the line. Describe the shape the set of midpoints of all of the line segments would form. Draw a sketch to accompany your conclusion.



5. *Math Concepts:* Open the **Math Laboratory Investigation 11.1: Read Me—Paper Folding Instructions** from the companion website and investigate the triangle constructions as described in 1, 2, 3, and 4 of the *Starting Points for Investigations 11.1*. Describe your investigation and explain your thinking.

6. *NCTM Standards:* Read over the **Geometry Standards** for all grade levels in the back pages of this book; list any *Expectations* that you believe are satisfied by activities that ask students to describe sets of points whose location is determined by given conditions. State the *Expectations,* the *Standards,* and the grade levels they are under.

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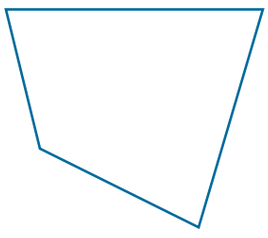
Follow-Up Questions and Activities 11.2

1. *School Classroom:* Make a list of the geometric knowledge and skills that you believe elementary school students will learn by successfully creating a translation tessellation. What can you add to your list if students also successfully create rotation tessellations and glide reflection tessellations? Explain in detail.

2. *School Classroom:* Find and copy at least two Escher-type drawings that you believe would be appropriate to discuss with elementary students. Accompany the drawings with a list of questions you could pose to an elementary class to begin a discussion of the mathematical aspects of the drawings. Explain what you would like the students to notice.

3. *Math Concepts:* Every quadrilateral tessellates the plane. However, can an arbitrary quadrilateral such as the one shown here have all its sides altered and still tessellate the plane? Decide which methods described in this section you can use to alter the sides of this quadrilateral and tessellate the plane. In the pictured quadrilateral, no sides are of equal length and no sides are parallel. For each method you use, make a template for your figure and determine whether or not it will tessellate the plane. Describe your results and include any clarifying diagrams.



4. *Math Concepts:* Go to the companion website and launch the Tessellation Applet in Chapter 11. Choose one of the tessellations discussed in this section and create it online. Print your results.

5. *Math Concepts:* One way to create a shape for tessellation is to use tracing paper to trace and reproduce the transformed edges as demonstrated in activity 1. Another way is to use an index card or cardstock paper, design how you will alter an edge, cut off the design, and then tape that piece appropriately on another edge as shown in figure 1.

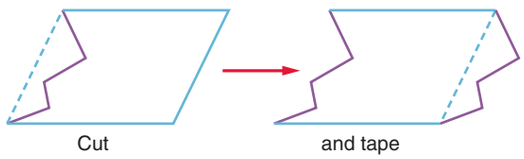


Figure 1

Figure 2

   a. Which tessellation-shaping techniques described in this activity set can be accomplished with this cut and tape process? Explain your thinking.
   b. Alter an isosceles trapezoid like the trapezoid shown in figure 2 and use the cut-and-tape method to form a template. Use this to form an Escher-type tessellation; sketch enough of the beginnings of a pattern so the reader has sufficient information to extend your pattern.

6. *NCTM Standards:* In the **Standards Summary** in the back pages of this book there are five **Process Standards.** The **Process Standards** discuss ways of acquiring and using content knowledge. Describe in detail how an activity in this section addresses an *Expectation* in one of the **Process Standards.** State the *Expectation* and the *Standard*.

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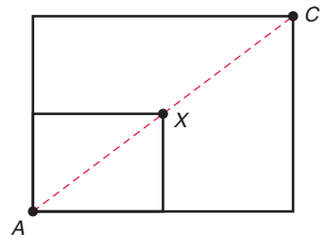
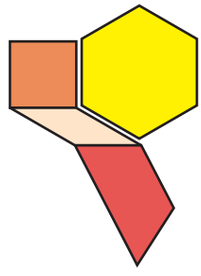
Follow-Up Questions and Activities 11.3

1. *School Classroom:* One of your students asks you if all squares are similar. Explain how you would explore this idea with your student.

2. *School Classroom:* A student says that a way to test whether or not two rectangles are similar is to put the smaller rectangle on the corner of the larger rectangle (as shown in the figure). Then, if the corner X on the smaller rectangle is on the diagonal AC of the larger rectangle, the two rectangles are similar. Explain whether or not this student is correct and why.

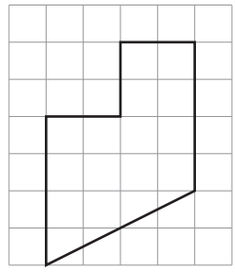


3. *Math Concepts:* Use your pattern blocks to form figure 1 below. Then use pattern blocks to form a new figure similar to figure 1 whose sides are in a ratio of 2 to 1 to figure 1. Sketch your resulting figure.



(1)

4. *Math Concepts:* On Centimeter Grid paper draw a new figure, similar to figure 2 below, so that the ratio from the sides of figure 2 to the sides of the new figure is 2 to 3. Compare the area of the new figure to the area of figure 2; how many times larger is the area of the new figure than figure 2? Centimeter Grid paper is available for download at the companion website.



(2)

5. *NCTM Standards:* Give an example of an activity for exploring similarity for Grades 3–5. What *Expectations* and *Standards* does your activity address? State the *Expectations* and *Standards* and explain why your activity addresses these recommendations.

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