Math 212, Final Exam Review

Math 212 Final Exam is an in-class exam given on Monday, December 8, 2014

THE FINAL TIME FOR ALL SECTIONS IS 12 – 1:50 p.m.

<u>LOCATION: HWC 105</u>

- The Final Exam is a mix of multiple choice and short answer questions.
- You may use your personal manipulative kit during the exam.
- You will need to use a calculator during the exam.
- You may not use a cell phone during the exam.
- You may use two 3” x 5” note cards on this exam with notes on both sides.

- ***The final is comprehensive and covers §5.2-§5.3, §6.1-§6.4, §7.1-§7.3, §8.1-§8.2***

PRACTICE PROBLEMS & PRACTICE PROBLEM IDEAS (THIS IS *NOT* MEANT TO BE AN EXHAUSTIVE LIST OF ALL THE TYPES OF PROBLEMS THAT MAY SHOW UP ON THE FINAL – JUST A SAMPLE.)

1. Sketch a fraction bar model showing $\frac{2}{3} \times \frac{3}{4}$

2. Sketch a fraction bar model showing $3 \div \frac{2}{3}$

3. Three people are sharing a bag of donuts. The first person takes $\frac{1}{4}$ of all the donuts in the bag. The second person takes $\frac{2}{3}$ of the remaining donuts. Then the third person takes $\frac{1}{2}$ of what is left. There are now 3 donuts left in the bag. How many were there to start?

4. Find a fraction between $\frac{1}{3}$ and $\frac{1}{2}$. Illustrate all three fractions with fraction bars.

5. Sketch fractions bars to illustrate $\frac{5}{6} - \frac{3}{4}$.

6. Write the following as a fraction or explain why you cannot. Show your work. Use the procedure we learned in class. Do not skip any steps:
   a. 0.1313…
   b. 0.351313…

7. For the following fractions that CAN be written as ***terminating decimals***; write each fraction as a decimal. Show your work. Use the procedure we learned in class. Do not skip any steps. For the following fractions that CAN NOT be written as a ***terminating decimal***; explain why this is the case.

   a. $\frac{1}{250}$ b. $\frac{1}{12}$ c. $\frac{6}{30}$

8. You have two candles. One is blue and 8 inches tall and the other is yellow and 12 inches tall. The blue candle burns $\frac{1}{4}$ of an inch every hour and the yellow candle burns $\frac{1}{2}$ inch every hour. If the yellow candle is lighted 6 hours after the blue candle and both candles burn continuously, which candle will burn out first? After the first candle has burned out, how much longer will the other candle burn?

9. Sketch a decimal square model to illustrate 1.2 x 1.5

10. Sketch a decimal square model to illustrate 0.60 ÷ 0.15

11. The ratio of local students to out-of-state students in a given school is 4 to 3. If the school has 490 students, how many are out-of-state students? Show your work.

12. Use a decimal square model to answer each of the following questions.
    a. At the cyber-Monday sale, the TV was discounted by 45%. If the sale price was $750, what was the original price?
    b. There are 350 teachers in the district. 80% of the teachers work at rural schools. How many teachers work at rural schools?

13. The odds that I will pass my exam are 5:1. What is the probability I will pass?

14. In my bag of M&M's I have 24 brown, 6 red, 6 blue and 4 green. Make a pie chart showing the colors. Indicate the measure of each central angle to the nearest degree.

15. Consider the following set of 20 exam scores: 53, 60, 68, 70, 70, 72, 75, 76, 76, 76, 80, 81, 82, 85, 86, 86, 86, 90, 95, 99
    a. Find the mean, median, mode and standard deviation of the exam scores.
    b. Are any of the scores "rare"? Explain.
    c. Sketch a box and whisker plot of this data.
    d. Are there any outliers? Explain.
    e. Sketch a stem and leaf plot of this data

16. A coat is on sale for $35 at the 30% off sale. What was the original price of the coat?

17. Suppose the mean on an exam was 75 with a standard deviation of 2.8. Cheryl had a z-score of -2.5 for this exam. What was her exam score?

18. Make a sketch of graph that is skewed *left*. For this graph would you most likely expect the mean < median, mean = median, or mean > median? Explain.

19. The number of hours per sleep a college student gets per night follows a normal distribution with mean 7 hours and standard deviation 1 hour.
    a. What percentage of college students get more than 8 hours of sleep per night?
    b. What percentage of college students get between 5 and 8 hours of sleep per night?
    c. If you took a random sample of 150 college students, how many of them would you expect to get between 7 and 8 hours of sleep per night?

20. Suppose that the names of the days of the week are placed in a box and one name is drawn at random. What is the probability of drawing a day that starts with T? M? S? How about a T or an S? How about starts with T or less than 7 letters?

21. A jar has 1000 jelly beans, and you know that P(Blue) = 3/5 and P(Red) = 3/8. What is the probability that a jelly bean drawn at random is neither blue nor red? P(not Blue)?

22. A box contains 3 red balls, 5 black balls, and 4 white balls. Suppose that **ONE** ball is drawn at random. Find each of the following: Probability that a black ball is drawn, P(B), how about P(R), P(W), P(not W), P(R or B). Compute the odds in favor of each of the previous.

23. A box contains 3 red balls, 5 black balls, and 4 white balls. Suppose that **TWO** balls are drawn at random (without replacement). Find each of the following: Probability that a black ball and then a white ball is drawn, P(one black and one white), how about P(at least one red), P(yellow), P(2 black)?

24. Assume that a family wants to have 4 children; list all of the outcomes in the sample space and then compute the probabilities of each possible outcome (4 girls, 4 boys etc.). Then compute P(at most 2 girls) and P(at least 2 girls). What is the expected number of girls?

25. You flip a coin 4 times. What is the probability of getting at least one head?

26. I have 8 sweaters and I want to bring 4 of them on my trip. How many different ways are there to do this?

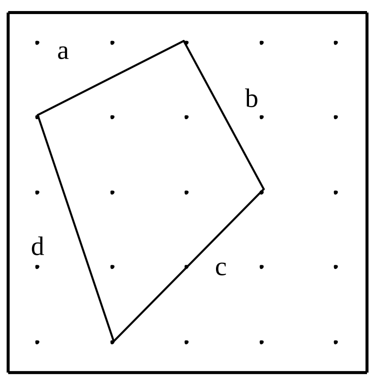
27. I have 8 sweaters that I want to give to 6 of my friends. How many different ways are there to do this?

28. There are 6 toppings and I want to pick 3 different toppings for my pizza. How many ways are there to do this?

29. In a game of chance, you roll a die and flip a coin. If the coin comes up heads you win the number of dollars shown on the die; if the coin comes up tails you pay the number of dollars shown on the die. (For example if you roll a 5 and flip a head you win $5; if you roll a 4 and flip a tail you pay $4.) What is the expected value of the game?

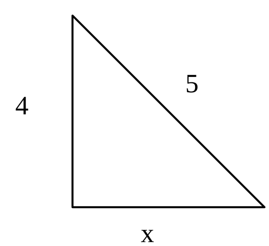
30. Determine if the following are rational or irrational. In each case simply so the lowest number is under the radical.
    a. $\sqrt{40}$
    b. $\sqrt[3]{16}$
    c. $\sqrt{169}$

31. Find the perimeter and area of the following figure:

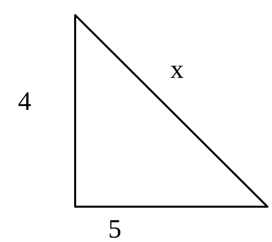


32. Find the length, x, of the missing side of the triangles:

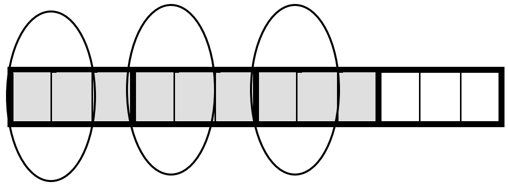
a.



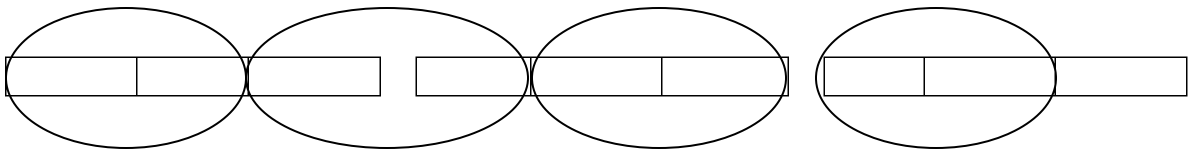
b.



Final Exam Review Answers

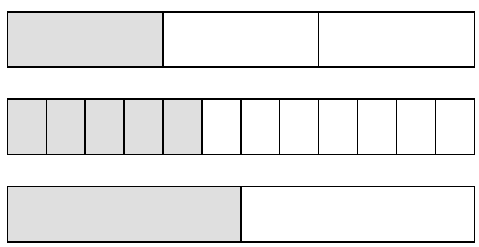
1. 

$$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$$

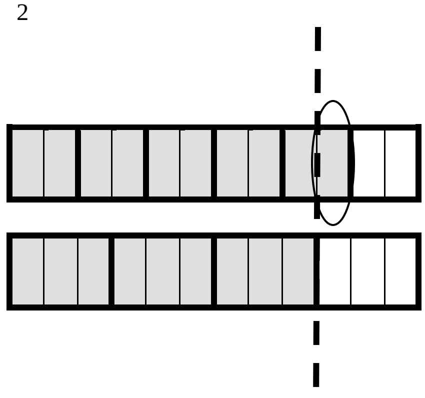
2. 

I can circle 4 groups of size $\frac{2}{3}$ with $\frac{1}{2}$ of a group left over so $3 \div \frac{2}{3} = 4\frac{1}{2}$.

3. 24 donuts
4. There are several possible answers – here is one:



$\frac{5}{12}$ The above illustrates $\frac{1}{3} < \frac{5}{12} < \frac{1}{2}$

5. 

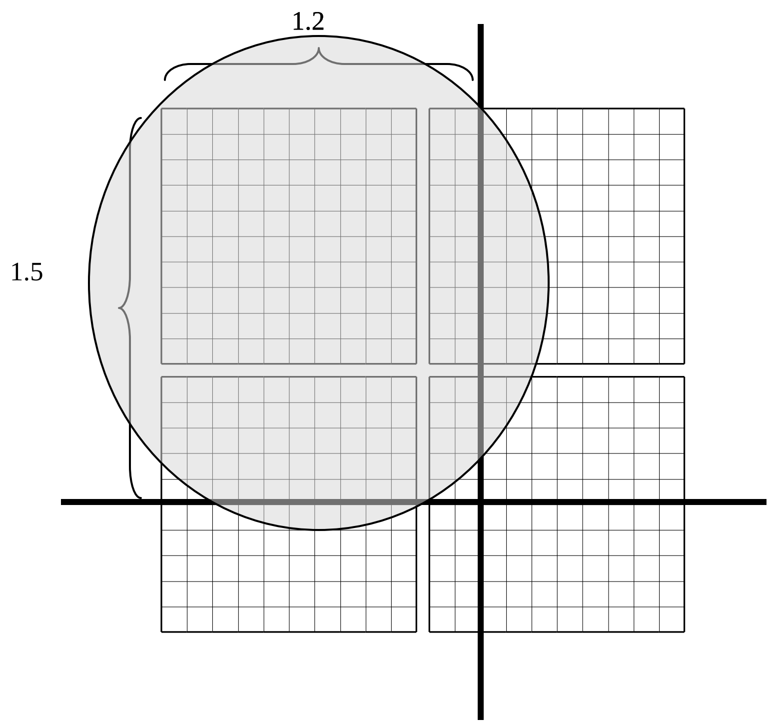
$$\frac{5}{6} - \frac{3}{4} = \frac{1}{12}$$

6. (a) $\frac{13}{99}$ (b) $\frac{3478}{9900}$

7. (a) $\frac{1}{250} = \frac{4}{1000} = 0.004$ (b) $\frac{1}{12}$ cannot be written as a terminating decimal because the denominator contains a factor other than 2 or 5. (c) $\frac{6}{30} = \frac{1}{5} = \frac{2}{10} = 0.2$

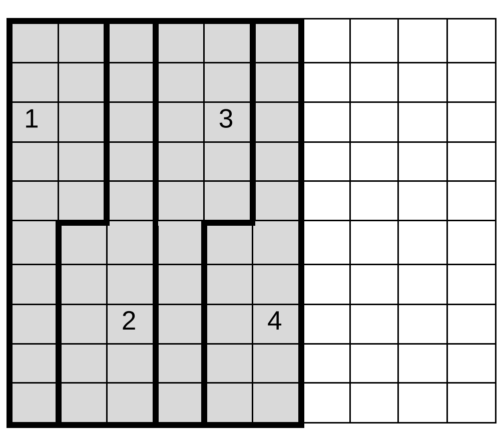
8. The yellow candle will burn out first; the blue candle will burn 2 more hours after that.

9.

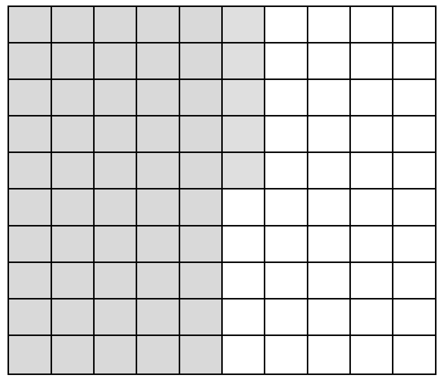


$1.2 \times 1.5 = 1.8$ (count up all the squares – you have one whole square = 1 and 80 small ones)

10. $0.60 \div 0.15 = 4$

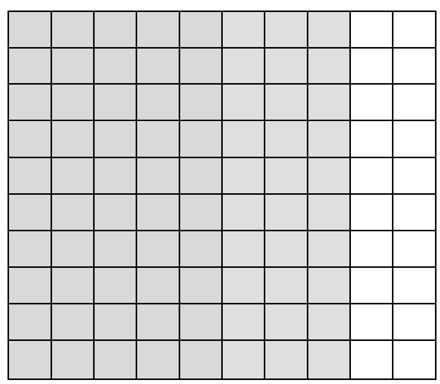


11. 210 out-of-state students
12. (a) ~$1364



The shaded area represents the sale price (55% of the original) which is $750. So 1 square = $750/55 ~ $13.64 so the original price (100%) is $13.64 x 100 =$1364.

(b) 280 teachers work at rural schools

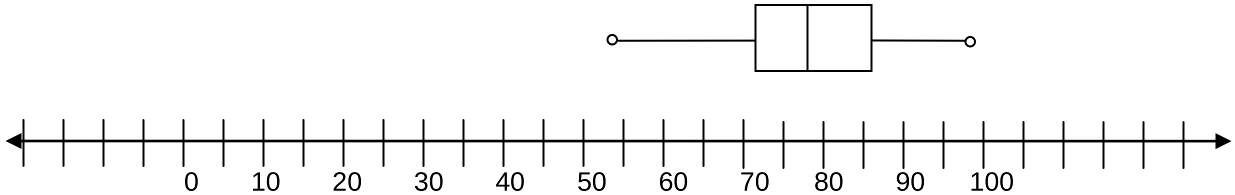


The entire square represents the 350 teachers in the district so 1 square = 350 teachers / 100 = 3.5 teachers. We want 80% = 80 squares which represents 80 x 3.5 = 280 teachers.

13. $\frac{5}{6}$

14. The central angles should measure: Brown: 216˚, Red: 54˚, Blue: 54˚, Green: 36˚

15. (a) mean = 78.3, median = 78, there are two modes: 76, 86, standard deviation = 10.9
(b) a ”rare” score is 2 or more standard deviations away from the mean (z-score greater than or equal to 2 or less than or equal to -2) 53 is rare since its z-score is -2.32.
(c) (53,71,78,86,99)



(d) The interquartile range is 86-71 = 15. Outliers are points more than 1.5 * 15 = 22.5 above the right edge of the box or below the left edge of the box; i.e. x < 71 – 22.5 = 48.5 or x > 86+22.5 = 108.5. There are no outliers.
(e)

Tens	Ones
5	3
6	08
7	0025666
8	0125666
9	059

16. $50
17. 68
18. In a skewed left graph the tail goes to the left. In general the tail is in the direction of the skew. We'd expect the mean to be "pulled down" from the middle by these low outliers thus making mean < median. See page 496 in your book for some pictures.
19. (a) 16% (b) 81.5% (c) 51 students
20. $\frac{2}{7}, \frac{1}{7}, \frac{2}{7}, \frac{4}{7}, \frac{5}{7}$
21. P(not blue and not red) = $\frac{1}{40}$, P(not blue) = $\frac{2}{5}$
22. $\frac{5}{12}, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}; 5:7, 1:3, 1:2, 2:1, 2:1$
23. $\frac{5}{33}, \frac{10}{33}, \frac{5}{11}, 0, \frac{5}{33}$
24. {GGGG BGGG GBGG GGBG
GGGB BBGG BGBG BGGB
GBBG GBGB GGBB GBBB
BGBB BBGB BBBG BBBB}

The corresponding probabilities are all $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$, so P(at most two girls (0,1 or 2 girls)) = $\frac{11}{16}$, and P(at least two girls(2,3, or 4 girls)) = $\frac{11}{16}$

Expected number of girls: $0 * \frac{1}{16} + 1 * \frac{4}{16} + 2 * \frac{6}{16} + 3 * \frac{4}{16} + 4 * \frac{1}{16} = 2$

25. P(at least one head) = 1 – P(0 heads) = $\frac{15}{16}$
26. ${}_8C_4 = 70$
27. ${}_8P_6 = 20160$
28. ${}_6C_3 = 20$

29.

Outcome value	Probability
H1 = +1	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T1 = -1	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
H2 = +2	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T2 = -2	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
H3 = +3	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T3 = -3	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
H4 = +4	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T4 = -4	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
H5 = +5	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T5 = -5	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
H6 = +6	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
T6 = -6	$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$

expected value:

$$1 \cdot \frac{1}{12} - 1 \cdot \frac{1}{12} + 2 \cdot \frac{1}{12} - 2 \cdot \frac{1}{12} + 3 \cdot \frac{1}{12} - 3 \cdot \frac{1}{12} + 4 \cdot \frac{1}{12} - 4 \cdot \frac{1}{12} + 5 \cdot \frac{1}{12} - 5 \cdot \frac{1}{12} + 6 \cdot \frac{1}{12} - 6 \cdot \frac{1}{12} = 0$$

30. (a) irrational; $2\sqrt{10}$ (b) irrational; $2\sqrt[3]{2}$; (c) rational; 13
31. Area = 6.5 units^2; perimeter = $2\sqrt{2} + 2\sqrt{5} + \sqrt{10}$ units
32. (a) x=3; (b) x=$\sqrt{41}$