Quick Notes about Exam 2

- There will be a short (10-20 min) non-calculator, no note card portion for factoring, exponents, order of operations, GCF, and LCM.
- You do not need to memorize ancient numerals. If needed, those will be provided.
- Make sure you can explain how the rectangular models explain the paper-pencil algorithms

Question 1

Without a calculator, find the prime factorizations of

- 3360
- 4520

Question 1 Solution

- $3360 = (2)^5(3)(5)(7)$
- $4520 = (2)^3(5)(113)$

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[0] 1:30

Question 2

Find the

- $GCF(3360, 4520)$
- $LCM(3360, 4520)$

Question 2 Solution

- $GCF(3360, 4520) = 40 = (2)^3(5)$
- $LCM(3360, 4520) = 379,680 = (2)^5(3)(5)(7)(113)$

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[0] 1:30

Question 3

Without using a calculator, determine

$$1 + 7 \times \left(3^5 \div 3^3 + 1\right) - 8 \div 2$$

Question 3 Solution

$$1 + 7 \times \left(3^5 \div 3^3 + 1\right) - 8 \div 2 = 67$$

[0] 1:30

Question 4

Give an example of a number with exactly 8 factors

Question 4 Solution

Multiple answers. Easiest way to create a number with exactly 8 factors is to use a number that has prime factorization with exactly three different primes each used once.

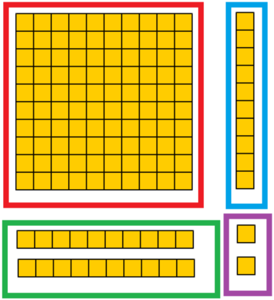
[0] 4:00

Question 5

Sketch the multiplication 12×11 using base 10 pieces in the array model, then show the corresponding four-partial products and how they relate to your sketch.

Question 5 Solution

Given is the array with the four-partial products



10 x 10 = 100

10 x 1 = 10

2 x 10 = 20

2 x 1 = 2

12 x 11 = 100 + 10+ 20 +2 = 132

[0] 3:00

Question 6

The Senior Square Dancers have dances every night of the week. Milly goes every 5^{th} night, Pauline goes every 6^{th} night, and Elmer goes every 10^{th} night. If all three are there on Tuesday night, how many days until they will all be at the dance together again?

Question 6 Solution

It will be 30 days until they will all be at the dance together again.
We used the $LCM(5, 6, 10)$ to find the solution. The $LCM(5, 6, 10) = 30$

[0] 2:00

Question 7

How many units are in 3201_{five}?

Question 7 Solution

There are 426 units in 3201_{five}
Since $3201_{five} = 3(5)^3 + 2(5)^2 + 0(5) + 1(1)$ and
$3(5)^3 + 2(5)^2 + 0(5) + 1(1) = 426$.

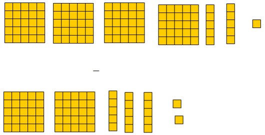
[0] 4:00

Question 8

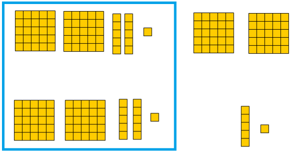
Sketch the base five pieces corresponding to the problem $421_{five} - 232_{five}$. Show all regroupings. Write the answer as a base five numeral.

Question 8 Solution

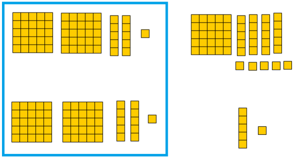
This is using the comparison model. The pieces outside of the blue box in the fourth step represent our number. As a base five numeral the answer is 134_{five}.



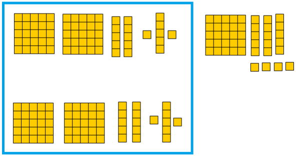
What we start with



Second Step: Group to together already like pieces



Third Step: Replace one flat with 4 longs and 5 ones



Fourth Step: Move Remaining like pieces into blue box

Question 9

Write 2785 in base 9

Question 9 Solution

$2785 = 3734_{nine}$.
Since $2785 = 3(729) + 7(81) + 3(9) + 4(1)$

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Question 10

What are the digits in base 9?

Question 10 Solution

0, 1, 2, 3, 4, 5, 6, 7, 8

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Question 11

Without using a calculator, determine if 2,345,678,920 is divisible by 6.

Question 11 Solution

2,345,678,920 is not divisible 6 since it is not divisible by 3.
Not divisible by 3 because $2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 2 + 0 = 46$ and $3 \nmid 46$.

Question 12

Are the whole numbers closed under subtraction? If not, give an example showing how this property fails.

Question 12 Solution

The whole numbers are not closed under subtraction if we try to subtract any two arbitrary whole numbers. For example $1 - 3 = -2$. -2 is not a whole number. Subtraction is closed if we require that we only subtract smaller numbers from larger numbers.

Question 13

Give an example of the commutative property for multiplication.

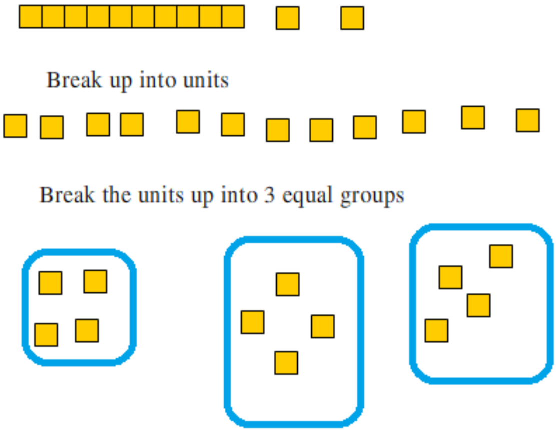
Question 13 Solution

$(2)(3) = 6$
$(3)(2) = 6$
So, $(2)(3) = (3)(2)$.
Any example which illustrates that order doesn't matter works.

Question 14

Make a sketch of $12 \div 3$ showing the "sharing" model of division

Question 14 Solution



Question 15

Make a word problem for $12 \div 3$ that demonstrates the "measurement" model of division.

Question 15 Solution

Anything that makes groups of 3 works.

Question 16

Consider the question "I have 72 pencils. My brother has 65. How many more pencils do I have?" What operation is being performed? Which concept of that operation does this model?

Question 16 Solution

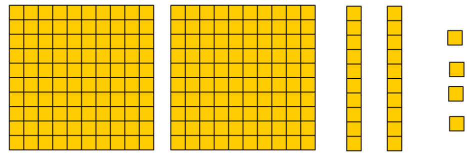
This is the comparison model of subtraction.

Question 17

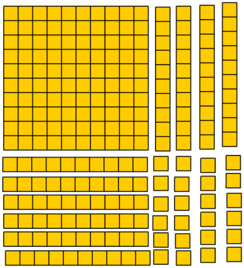
Use the array model of division for $224 \div 16$ using your base 10 pieces.

Question 17 Solution

We get $224 \div 16 = 14$



Need to turn this into a rectangle that has one side of length 16



Got a 16 by 14 rectangle by breaking 1 flat into 10 longs and 2 longs into 20 units

Question 18

How many factors does $(2)^3(3)^2(5)$ have?

Question 18 Solution

$(2)^3(3)^2(5)$ has 24 factors.
Note, $24 = (3+1)(2+1)(1+1)$

Question 19

Suppose A and B are whole numbers with $GCF(A, B) = 15$ and $AB = 2250$. What is $LCM(A, B)$?

Question 19 Solution

$$LCM(A, B) = \frac{2250}{15} = 150$$

Question 20

There are 150 blue M&Ms, 80 red M&Ms, and 120 brown M&Ms. If I want to put them into bowls so each bowl has the same number of each color of M&Ms (and each bowl contains all three colors), then what is the greatest number of bowls I can use? How many M&Ms of each color are in each bowl?

Question 20 Solution

I can use at most 10 bowls since $GCF(150, 80, 120) = 10$.
There will be 15 blue, 8 red, and 12 brown M&Ms in each bowl.

Question 21

If I have n tiles, and I can make 5 different non-square rectangles with these tiles, then how many factors does n have?

Question 21 Solution

10 factors since each non-square rectangle gives two factors.

Question 22

Answer True or False to the following

- Prime numbers have exactly 1 factor
- Odd numbers have an odd number of factors

Question 21 Solution

- Prime numbers have exactly 1 factor
  False prime numbers have exactly two factors
- Odd numbers have an odd number of factors
  False 15 is odd and 15 has factors 1, 3, 5 ,15.