

# Chapter 8

## Groundwater Flow

### 8.1 Aquifers and Porosity

In order that water (or any other liquid, such as oil) can flow through rock, it is necessary that the rock not only be porous, but also *permeable*. That is, the pores (or at least some of them) must form, in effect, a connected network of tiny pipes through which water can move. Figure (8.1a) shows porous but impermeable rock, and Fig.(8.1b) shows permeable rock.

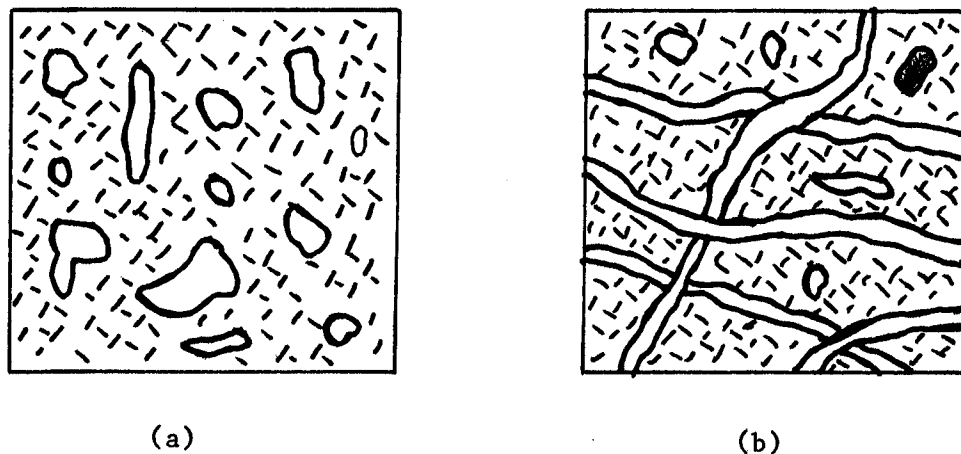


Fig.(8.1) Impermeable (a) and Permeable (b) Rock

Note that some of the pores in Fig.(8.1b) do not form part of the connected pore system, but are isolated from it. Any water contained in these pores is cut off from the system of connected pores. (Evidently, this water found itself trapped in the isolated pores that formed as the rock solidified.) The water in these isolated pores cannot flow to a well, say, or be pumped out of the rock. Rather, this water is retained by the rock.

Let  $V_{\text{retain}}$  be the total volume of the isolated pores (and hence of the retained water if the pores are filled), and  $V_{\text{yield}}$  the volume of all the connected pores (and hence the volume of the water the rock can yield if these pores are filled) in a certain block of rock. As in Chapter 1, the total pore volume is  $V_{\text{pores}}$ . The volume of the block of rock, as given from its external dimensions, is  $V$ .

The total pore volume  $V_{\text{pores}}$  is given by

$$V_{\text{pores}} = V_{\text{retain}} + V_{\text{yield}}. \quad (8.1)$$

Dividing by  $V$  gives

$$\frac{V_{\text{pores}}}{V} = \frac{V_{\text{retain}}}{V} + \frac{V_{\text{yield}}}{V}. \quad (8.2)$$

Now  $V_{\text{pores}}/V = n$ , the porosity of the rock; see Eq.(1.7). In a similar manner, define the *yield porosity*  $n_y$  by

$$n_y = \frac{V_{\text{yield}}}{V}, \quad (8.3)$$

and the *retention porosity*  $n_r$  by

$$n_r = \frac{V_{\text{retain}}}{V}. \quad (8.4)$$

Therefore, by Eq.(8.2),

$$n = n_r + n_y. \quad (8.5)$$

The porosity that "counts" in groundwater flow is the yield porosity  $n_y$ , sometimes called the *effective porosity*. The isolated pores, and any water they contain, are "de facto" part of the rock matrix.

A formation, or layer, of permeable rock is called an *aquifer*, a layer of impermeable rock an *aquiclude*.

An aquifer bounded only from below by an aquiclude is termed an *unconfined aquifer*; an aquifer bounded both from below and above by an aquiclude is a *confined aquifer*; see Fig.(8.2). Only unconfined aquifers are considered in the present work.

As indicated in Chapter 5, the water table, symbol  $\nabla$ , is the surface below which the pores are completely filled with water (saturated rock or soil) and above which the pores are empty (dry rock or soil). Usually there is a finite transition region between the saturated and dry layers, but it will be assumed that this transition region is so thin that it can be

considered a discontinuity.

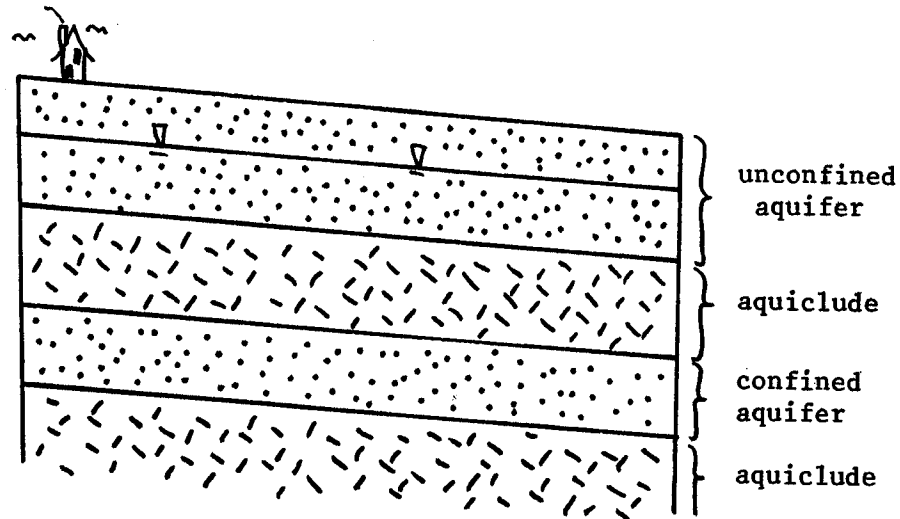


Fig.(8.2) Aquifers

## 8.2 Aquifer Discharge

Consider a rectangular block of aquifer rock below the water table. The block has horizontal width  $w$ , vertical thickness  $h$  and horizontal (or near horizontal) length  $L$ . Suppose that water is moving through the rock in the direction parallel to the edges of length  $L$  and perpendicular to the cross section with edge lengths  $w$  and  $h$ . Since the water can move only through the connected pores, the discharge  $Q$  of the water through the cross section is

$$Q = A_{\text{pores}} v, \quad (8.6)$$

where  $A_{\text{pores}}$  is the total cross-sectional area of all the connected pores in the aquifer cross section, and  $v$  is the speed of the water in the pores as the water passes the cross section. Note that Eq.(8.6) implies the same definition of discharge as for the discharge of a river:  $Q = V/t$ , where  $V$  is the volume of water passing through a cross section in time  $t$ .

The quantity  $A_{\text{pores}}$  is hardly ever measurable directly in the field. Rather, it is the cross-sectional area  $A$  of the saturated part of the aquifer itself that can be more easily determined, where  $A = wh$ . It is convenient to express Eq.(8.6) in terms of  $A$  rather than  $A_{\text{pores}}$ .

To relate  $A$  to  $A_{\text{pores}}$ , imagine that the connected pores through which the water moves constitute tiny pipes extending the length  $L$  of the aquifer in the direction of flow. By definition of the yield porosity  $n_y$ , Eq.(8.3),

$$V_{\text{yield}} = n_y(Lwh). \quad (8.7)$$

The total volume of all the connected pores can now be expressed as  $V_{\text{yield}} = A_{\text{pores}}L$ . The saturated aquifer cross-sectional area is  $A = wh$ . Using these two equations, Eq.(8.7) becomes

$$\begin{aligned} A_{\text{pores}}L &= n_yAL, \\ A_{\text{pores}} &= n_yA, \end{aligned} \quad (8.8)$$

where  $A_{\text{pores}}$  is the total cross-sectional area of all the connected pores cutting the wetted aquifer cross section of area  $A$ .

Substituting the last equation into Eq.(8.6) for the discharge gives

$$Q = (n_yA)v. \quad (8.9)$$

Rearrange this to read

$$Q = A(n_yv),$$

and then define  $v_D$ , by

$$v_D = n_yv, \quad (8.10)$$

to get for the discharge the expression

$$Q = Av_D. \quad (8.11)$$

Equation (8.11) for the discharge now looks like the equation  $Q = Av$  for the discharge of a river, except that the real speed  $v$  of the water has been replaced with the quantity  $v_D$  defined by Eq.(8.10). Now  $v_D$  has units of speed, since  $n_y$  in Eq.(8.10) has no units, and it is called various names, prominent among them the *Darcy speed*, named after the nineteenth century engineer Henry Darcy. It is not the speed at which the water, or liquid, actually moves, but is only a mathematical construction with units of speed. Nevertheless,  $v_D$  is sometimes misleadingly called the "percolation speed" or even "flow velocity". The actual velocity at which the liquid moves is  $v$ , not  $v_D$ .

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#### EXAMPLE 1

An aquifer has a cross section with a horizontal width of 265 m, and a vertical thickness below the water table of 42.0 m. The water table is 36.0 m below the ground surface. Each day, 3340 m<sup>3</sup> of water is discharged through the cross section. The aquifer rock has an effective porosity of 27.1%. Find (a) the Darcy speed, and (b) the actual speed of the water as it passes through the aquifer.

(a) The discharge  $Q$  can be calculated from

$$Q = \frac{V}{t},$$

$$Q = \frac{3340 \text{ m}^3}{86,400 \text{ s}},$$

$$Q = 0.03866 \text{ m}^3/\text{s}.$$

The cross sectional area  $A$  of the aquifer through which the water is discharged is

$$A = wh,$$

$$A = (265 \text{ m})(42 \text{ m}),$$

$$A = 1.113 \times 10^4 \text{ m}^2.$$

For  $h$ , use the vertical thickness of that part of the aquifer cross section that is below the water table; i.e.,  $h = 42 \text{ m}$ . There is no water above the water table. Now use Eq.(8.11):

$$Q = Av_D,$$

$$0.03866 \text{ m}^3/\text{s} = (1.113 \times 10^4 \text{ m}^2)v_D,$$

$$v_D = 3.47 \times 10^{-6} \text{ m/s}.$$

(b) The actual speed  $v$  of the water at discharge is given from Eq.(8.10). Convert the porosity to a decimal value, and then find that

$$v_D = n_y v,$$

$$3.47 \times 10^{-6} \text{ m/s} = (0.271)v,$$

$$v = 1.28 \times 10^{-5} \text{ m/s},$$

$$v = 0.0128 \text{ mm/s}.$$

Groundwater flow can be very slow.

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## 8.3 Darcy's Law

So far, the only parameter that has entered the analysis of groundwater flow is the yield porosity of the aquifer rock. No properties of the fluid itself have entered. But different fluids (water and oil, for example) have different densities and viscosities (internal friction), for example. Hence, different fluids could be expected to flow at different speeds through the same aquifer. Nothing in the equations so far presented would allow for such a difference; hence, the set of equations developed to this point must be incomplete.

Additional equations can be sought by asking why should water (or other liquid) flow through the aquifer at all, or why should it not simply sit in place in the pores of the rock (as, indeed, sometimes it does).

Consider an aquifer in which the water table is horizontal everywhere. In this configuration, the water will just sit in the connected pores; there is no tendency for the water to move in any horizontal direction. The reason is that gravity acts vertically down and has no horizontal component. Since the water table is horizontal, there is no component of gravity parallel to the water table.

In the absence of applied external pressure, water will flow through an unconfined aquifer for the same reason that water flows in a river. Water flows downhill under gravity.

In an aquifer in which the water table is inclined at the angle  $i$  with the horizontal, gravity  $g$  has the component  $g \sin i$  parallel to the water table. Water will move in the direction of this component of gravity in an attempt to bring the water table into a horizontal position.

Focus now on the speed of the water. If an object is released from rest on a frictionless table inclined at the angle  $i$  with the horizontal, the object's speed  $v$  after time  $t$  will be  $v = (g \sin i)t$ . That is, the speed continues to increase with time. But water, trying to move through tiny pores in rock is definitely not in a frictionless environment. In fact, there is so much friction, within the liquid and between the liquid and the inner surface of the pores, that the water moves at constant speed, the driving force of gravity being balanced by the resisting friction forces.

It is extremely difficult, even more difficult than the case of water flowing in an open channel, to derive a formula for this constant speed  $v$  from theory alone, so reliance is put on an empirical equation suggested by experiments. The engineer Henry Darcy, who is mentioned above, conducted experiments on the flow of liquids through porous materials. He obtained an empirical formula (now called Darcy's law, naturally enough) that describes his observations.

The water table is presumed to be inclined at the angle  $i$  with the horizontal, and it is presumed that  $i$  is not too large. (If the angle is too large, the speed of the water may reach values so that turbulence sets in, and Darcy's law is not valid under such circumstances.) As a result of being inclined, the water table drops a vertical distance  $H$  over a horizontal distance  $L$ . Darcy's law states that the Darcy speed  $v_D$  is given by

$$v_D = K \left( \frac{H}{L} \right). \quad (8.12)$$

The quantity  $K$  is called the *hydraulic conductivity*. Since the units of  $H$  and  $L$  in Eq.(8.12) cancel,  $K$  is seen to have the same units as  $v_D$ ; i.e., m/s in SI base units. The numerical value of  $K$  depends on the properties of the rock and on properties of the liquid. Specifically, for the rock it is the degree to which the pores are connected, and their size, that enters into  $K$ . For the liquid, the viscosity (internal friction) and density affect the value of  $K$ . This means that for any one given kind of rock, there are different values of  $K$  for water and for oil, for example.

Now, the movement of water through rock is usually a very slow process. The pores are very small, their geometry complex, and friction ever present. This means that the real flow speed  $v$  is usually very small. For this reason, hydraulic engineers often use the day (abbreviation d) rather than the second (s) as the unit of time. That is, values of  $v$ ,  $v_D$ , and

$K$  are generally expressed in m/d rather than the base units of m/s. For water, values of  $K$  for various rocks range between near zero and several hundreds of meters per day.

Equation (8.12) can be combined with Eq.(8.11) to yield this equation for the discharge:

$$Q = KA\left(\frac{H}{L}\right). \quad (8.13)$$

Sometimes this equation, rather than Eq.(8.12), is referred to as Darcy's law.

Darcy's law is sometimes expressed in terms of the angle  $i$  of inclination of the water table. Since  $i$  is small (for Darcy's law to be valid),  $i \approx \tan i$  when  $i$  is expressed in radians. From Fig.(8.3) notice that  $\tan i = H/L$ . Hence, for small  $i$ ,  $i = H/L$ . Equation (8.13) can now be written as

$$Q = KAi. \quad (8.14)$$

Sometimes, this is called Darcy's law.

In whichever version it is written, Darcy's law provides the additional equation needed to describe the flow of water (or another liquid) through permeable rock. The quantity  $i$  (or  $H/L$ ) is often called the *hydraulic gradient*. Also, the parameter  $K$  is sometimes called the *coefficient of permeability*. As mentioned in several places in the present work, engineering nomenclature is not standardized.

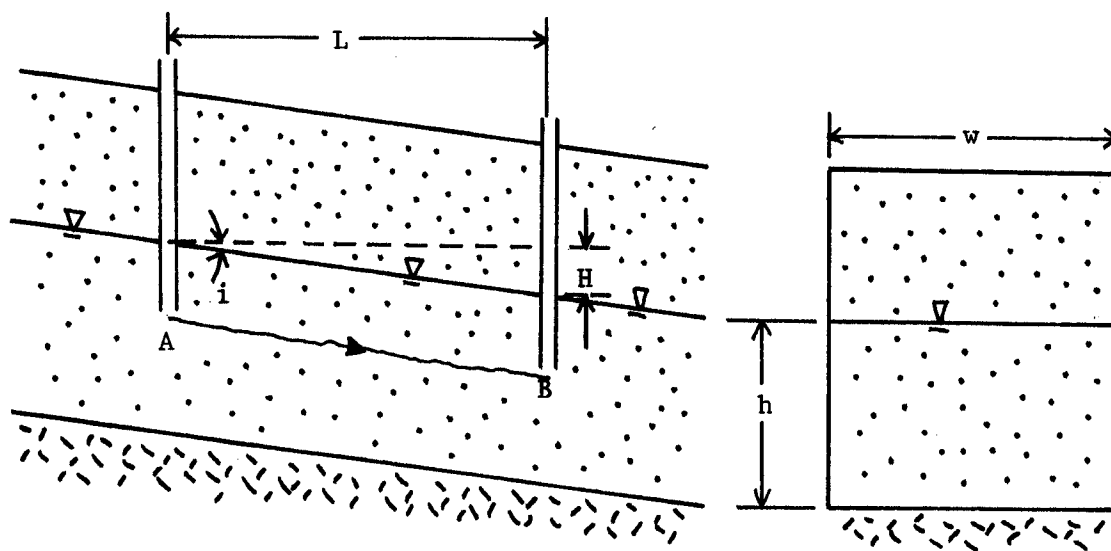


Fig.(8.3) Setting of an Unconfined Aquifer

Figure (8.3) shows an idealized setting of a typical unconfined aquifer. The uphill source of the groundwater, precipitation over a certain land area, is not shown. The ground surface slopes down "gently" (i.e., at a small angle  $i$ ) from left to right. Usually, the water table is roughly parallel to the ground surface, falling a vertical distance  $H$  over a horizontal distance  $L$ . Here, the additional assumption is made that the layer of impermeable rock beneath the aquifer also slopes down at the angle  $i$ . This means that the vertical thickness

$h$  of the water-bearing part of the aquifer has everywhere the same value. Two wells have been driven into the aquifer, piercing the water table at points A and B. The time  $t$  needed for the groundwater to move from A to B can be measured by dumping marked water into the well at A and noting when it shows up at well B. (Water can be marked with dye, or with a small quantity of radioactive heavy water.) Strictly speaking, the travel time from A to B is given by

$$t = \frac{AB}{v}.$$

Now

$$AB = \sqrt{L^2 + H^2}.$$

Since  $i$  is small,  $H^2 \ll L^2$ , so that  $AB \approx L$ . This approximation is appropriate not only because the hydraulic gradient is small, but because the values of the hydraulic conductivity and effective porosity are not likely to be known with great precision throughout the aquifer. Therefore, it is sufficiently accurate to use

$$t = \frac{L}{v}. \quad (8.15)$$

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### EXAMPLE 2

The longitudinal and transverse cross sections of an unconfined aquifer are shown in Fig.(8.4). Water takes 1.91 y to move from well A to well B. The hydraulic conductivity of the aquifer rock is 135 m/d. (a) Find the yield porosity of the aquifer rock. (b) It is found that  $8.42 \times 10^5 \text{ m}^3$  of water passes through any cross section of the aquifer in 2.00 weeks. Find the width  $w$  of the aquifer.

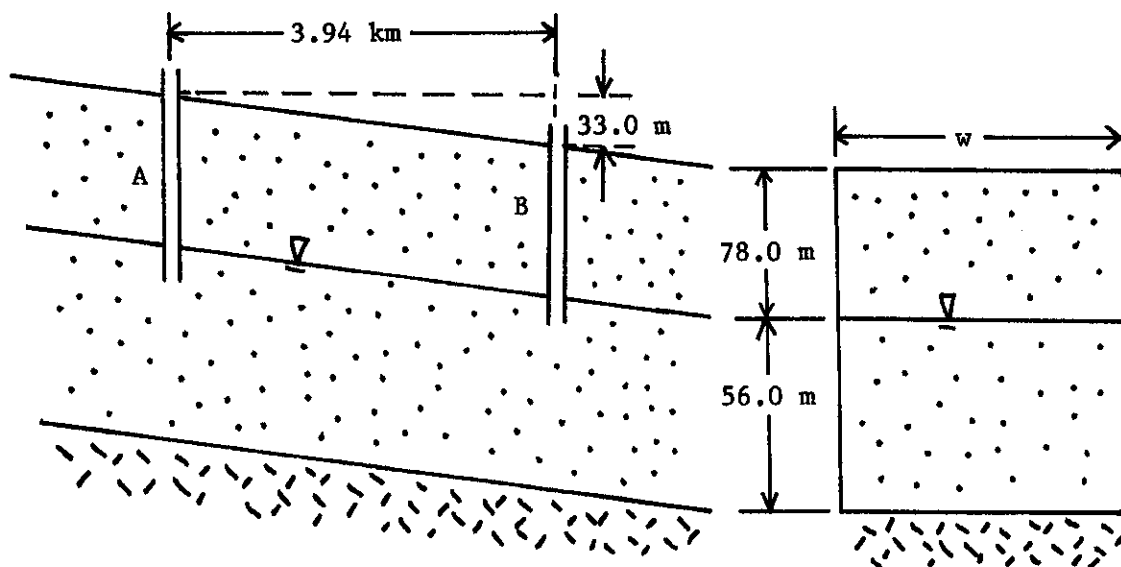


Fig.(8.4) Example 2



(a) Reading distances from Fig.(8.4), the data are:  $L = 3940$  m,  $H = 33$  m (not 78 m: the water table is parallel to the ground surface, and therefore drops by the same distance as does the ground surface);  $h = 56$  m (thickness of aquifer below the water table). Also,  $t = (1.91 \text{ y})(365.25 \text{ d/y})$ ,  $t = 697.6$  d;  $K = 135$  m/d. By Eq.(8.12), the Darcy speed is

$$v_D = K\left(\frac{H}{L}\right),$$

$$v_D = (135 \text{ m/d})\left(\frac{33 \text{ m}}{3940 \text{ m}}\right),$$

$$v_D = 1.131 \text{ m/d}.$$

The actual speed of the water follows from Eq.(8.15):

$$v = \frac{L}{t},$$

$$v = \frac{3940 \text{ m}}{697.6 \text{ d}},$$

$$v = 5.648 \text{ m/d}.$$

By Eq.(8.10),

$$v_D = n_y v,$$

$$1.131 \text{ m/d} = n_y (5.648 \text{ m/d}),$$

$$n_y = 0.200 \text{ (20.0\%)}.$$

(b) Evidently, the discharge is

$$Q = \frac{V}{t},$$

$$Q = \frac{8.42 \times 10^5 \text{ m}^3}{14.0 \text{ d}},$$

$$Q = 6.014 \times 10^4 \text{ m}^3/\text{d}.$$

Note that the symbol  $t$  for time in this part is not the time for the water to move from A to B. It would be tedious to use two different, or subscripted, symbols for various time intervals. As usual in such situations, the context of the discussion reveals the appropriate identification for  $t$ . Now apply Eq.(8.11) with  $A = wh$ , so that

$$Q = whv_D,$$

$$6.014 \times 10^4 \text{ m}^3/\text{d} = w(56 \text{ m})(1.131 \text{ m/d}),$$

$$w = 950 \text{ m}.$$

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## 8.4 Problems

- ① A rock has a porosity of 43.6%, but 35.0% of the pores are isolated. Calculate the yield porosity of the rock.
- ② Write an equation equivalent to Eq.(8.3), but in which the yield porosity is obtained as a percent.
- ③ The aquifer shown in Fig.(8.5) has a width of 7.10 km. Industrial waste BXC is illegally dumped into well A, near the factory. This material moves with the ground water, and appears at well B after 6.40 y. The effective porosity of the aquifer rock is 0.350. Calculate (a) the hydraulic conductivity of the aquifer rock, and (b) the aquifer discharge.

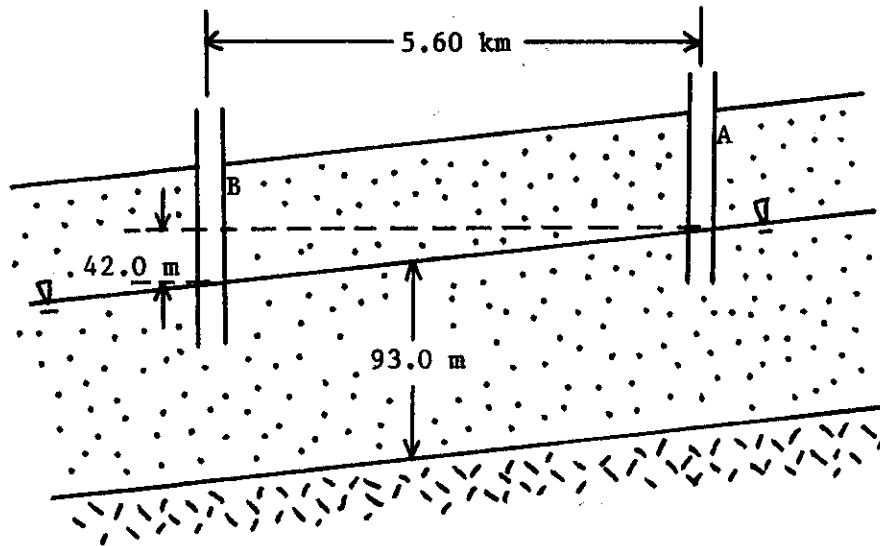


Fig.(8.5) Problem 3

- ④ An aquifer in a certain region consists of coarse sand with a yield porosity of 39.0% and a hydraulic conductivity of 45.0 m/d. The width of the aquifer is 2.90 km and its vertical thickness below the water table is 73.0 m. The aquifer discharge equals 8380 m<sup>3</sup>/d. Calculate the vertical drop of the water table over a horizontal distance of 6.60 km.
- skip ⑤ Show that the hydraulic conductivity  $K$  of the rocks of an unconfined aquifer like that shown on Fig.(8.4) can be expressed as

$$K = \frac{vv_D}{v_i}$$

where  $v_D$  is the Darcy speed,  $v$  the true speed of the groundwater, and  $v_i$  the vertical speed of the water induced by the slope of the water table.

- (6.) Find the porosity of an aquifer rock with a retention porosity of 23.3%, and through which the speed of the water is 3.40 times the Darcy speed.
- (7.) The aquifer shown on Fig.(8.6) has a width of 1.70 km. Water takes 2.50 y to move from the upper well to the lower. The hydraulic conductivity of the aquifer is 31.0 m/d. Find (a) the yield porosity of the aquifer, (b) the volume of water that passes any cross section in 3.00 weeks, and (c) the total volume of connected pore space in the aquifer between the wells and above the water table.

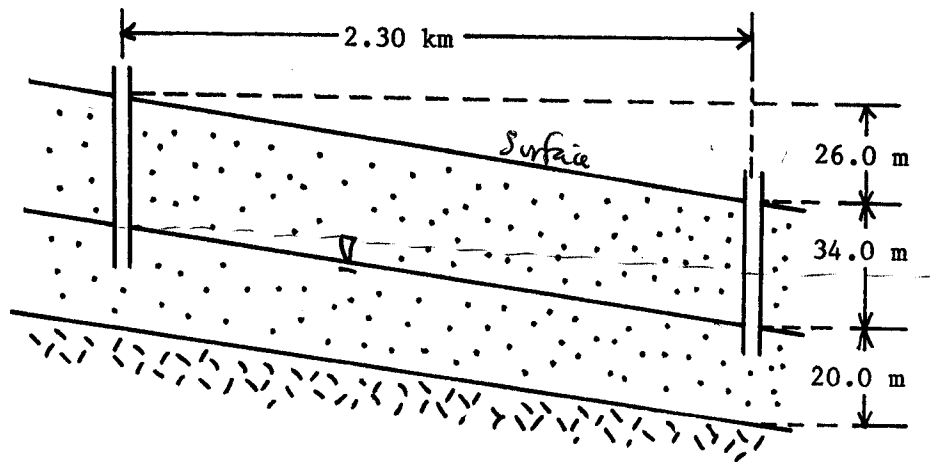


Fig.(8.6) Problem 7

- (8.) Consider the aquifer shown in Fig.(8.7); the aquifer is 830 m wide. The yield porosity of the medium is 47.3% and its hydraulic conductivity equals 143 m/d. (a) How long does it take the groundwater to move from well A to well B? (b) How long does it take for 150,000 metric tons of water to pass through a cross section of the aquifer?

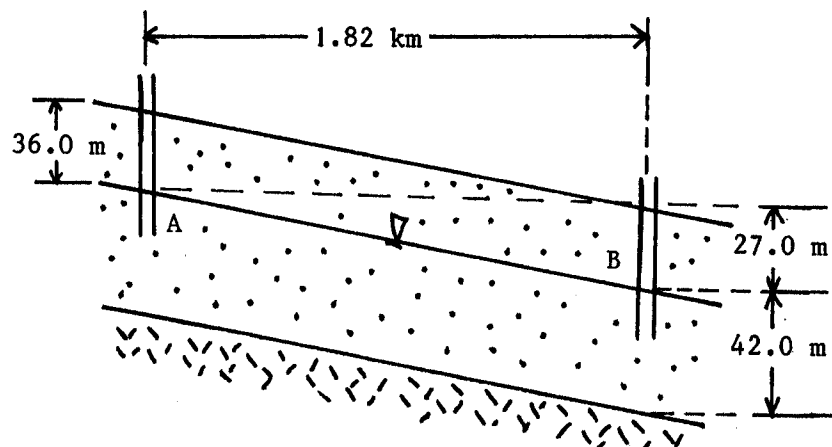


Fig.(8.7) Problem 8

9. The aquifer of Fig.(8.8) has a yield porosity of 34.0%, a retention porosity of 19.2% and a hydraulic conductivity of 160 m/d. (a) How long does it take for water to move from well A to well B? (b) What volume of water passes any cross section in 10.0 d? (c) Assuming that all pores below the water table are filled, find the total volume of water trapped in the rock between the wells.

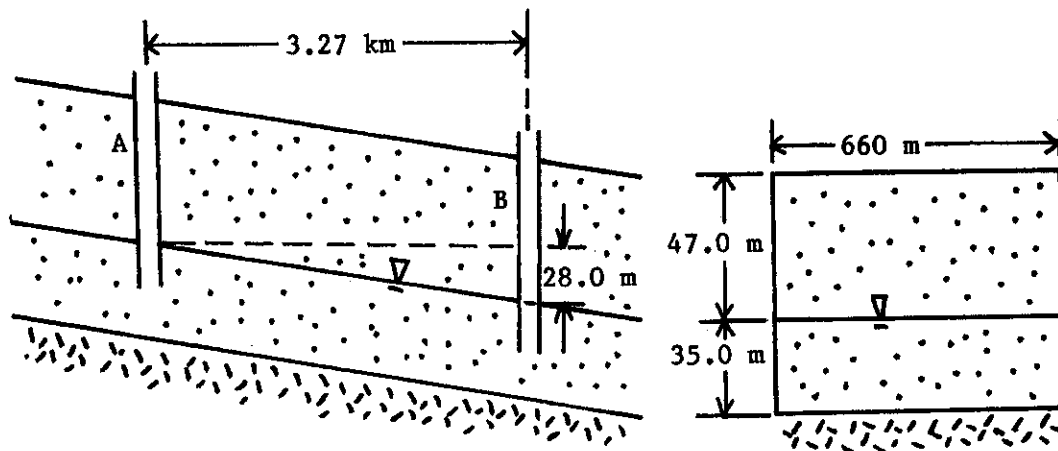


Fig.(8.8) Problem 9

10. The quantity of a chemical dissolved in groundwater is usually expressed either in units of mg/L (milligrams of chemical per liter of water), or as ppm, i.e., mass of chemical per million parts of mass of water. Show that the numerical values of these two quantities for a particular chemical are virtually the same.
11. Show that the time  $t$  required for groundwater to travel a horizontal distance  $L$  can be expressed as

$$t = \frac{n_y L^2}{KH},$$

where  $n_y$  is the yield porosity and  $K$  the hydraulic conductivity of the aquifer rocks, and  $H$  is the vertical drop of the water table over the horizontal distance  $L$ .

12. An unconfined aquifer extends to a depth of 73.0 m beneath a drainage basin of area 6.82 ha. The effective porosity of the aquifer rocks is 47.2%. Due to pumping of water, the water table falls from 12.6 m to 15.7 m beneath the ground surface. Find the volume of water pumped out of the ground.
13. An unconfined aquifer with width 420 m extends down to an impermeable rock layer at a depth of 85.0 m. The effective porosity of the aquifer rocks is 53.6%. When water moves

through the aquifer at a speed of 1.27 m/d, the aquifer discharge is found to be  $2.00 \times 10^7$  L/d. Find the depth to the water table.

14. Calculate the Darcy speed through an unconfined aquifer 870 m wide and 106 m thick when the discharge equals  $2.19 \text{ m}^3/\text{s}$  and the water table is 18.0 m beneath the ground.
15. Over a horizontal distance of 2.87 km, the water table falls by 41.5 m. Groundwater takes 3.18 y to move the 2.87 km. The effective porosity of the aquifer is 0.640, and the aquifer's cross-sectional area is 1.80 ha. Find the hydraulic conductivity of the aquifer rocks.
16. The aquifer shown in Fig.(8.9) is 1.30 km wide. The effective porosity is 0.290 and the hydraulic conductivity equals 83.0 m/d. Water containing a dangerous radioactive isotope of plutonium is dumped (accidentally, of course) into well A. How long does it take for the plutonium to show up in well B? Assume that the plutonium is carried along at the same speed as the groundwater.

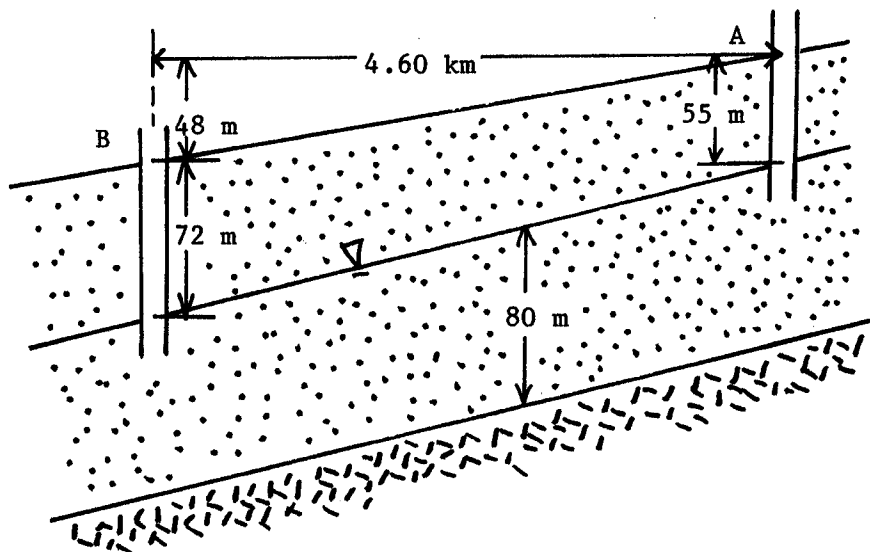


Fig.(8.9) Problem 16

17. Industrial waste containing the deadly contaminant XYX is dropped by terrorists into well A. Well B is 2.63 km from well A and at 55.0 m lower elevation. Between the wells, the depth to impermeable rock is 80.0 m, and the depth to the water table 16.7 m. The aquifer width is 1650 m. The aquifer rocks have a yield porosity of 0.364 and a hydraulic conductivity equal to 24.0 m/d. Assuming that XYX moves along with the groundwater, how long will it take for the contaminant to reach well B?

# ENV. Geology LAB - CONTAMINATION EQUATIONS

(1)  $V_p = V_r + V_{yield}$

$V_p$  = TOTAL VOLUME OF PORE SPACE

$V_r$  = VOLUME HELD RETAINED IN PORE SPACE (Porewater)

$V_{yield}$  = VOLUME HELD RELEASED FROM ADSORPTION

$V_t$  = TOTAL VOLUME ROCK

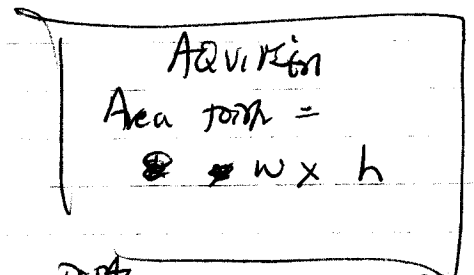
(2)  $n = n_y + n_r$   
 $n = porosity = V_p / V_t =$  TOTAL Porosity

(3) YIELD Porosity =  $V_y / V_t = n_y =$  EFFECTIVE Porosity

(4) Retention Porosity  $V_r / V_t = n_r$

(5) CONTINUITY EQUATION FOR ADSORPTIONS

$$Q = A_{pores} V$$



$Q$  = ADSORPTION DISCHARGE  
 $(L^3/t)$

$A_{pores}$  = TOTAL CROSS-SECTIONAL AREA OF PORES

$V$  = VELOCITY OF GROUNDWATER

(6)  $V_{yield} = n_y (Area)$  yield volume = yield porosity

x CROSS-SECTIONAL AREA

(7) Darcy velocity in SEEPAGE VELOCITY

$$V_D = n_y V$$

$V_D$  = seepage velocity

$n_y$  = yield porosity

$V$  = groundwater velocity

## Darcy's Law

$$Q = K I A$$

$Q$  = Discharge ( $L^3/t$ )

$K$  = hydraulic conductivity ( $L/t$ )

$I$  = hydraulic gradient (dimensionless)

$A$  = cross-sectional area of aquifer

## Darcy Speed

$$V_d = K \left( \frac{\Delta H}{L} \right) = K I$$