JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 91, NO. B1, PAGES 421-434, JANUARY 10, 1986

AN ANALYSIS OF THE ORIENTATIONS OF LARGE-SCALE CRUSTAL STRUCTURES: A STATISTICAL APPROACH BASED ON AREAL DISTRIBUTIONS OF POINTLIKE FEATURES

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Abstract. The spatial distributions of magmatic complexes and other features thought to be associated with major crustal structures may be important sources of information about large-scale structural patterns. However, attempts to incorporate these features into quantitative analyses of linear features have used arbitrary or inaccurate criteria to judge hypothetical geological relationships. In this paper, features of limited spatial extent are considered pointlike, and the concept of a probabilistic lattice point distribution is used to formulate a statistical method that leads to a quantitative and reproducible analysis of directional patterns based solely on the locations of the points. Thus this analysis is independent of linear patterns and provides a measure of the directional information intrinsic to point patterns. The procedure determines the most likely trends of structural anisotropies; Monte Carlo simulations of random point patterns provide a reference distribution from which confidence levels can be determined. Applications to published data for magmatic complexes, magnetic contour closures, and structural change points are used as examples. The results suggest that there has been a tendency to overestimate the amount of information available from point patterns.

Introduction

The orientations of linear features play an important role in identifying the large-scale structure of the continental crust. The majority of papers presented at several recent conferences (e.g., the Fifth International Conference on Basement Tectonics, Cairo, 1983) have been based on lineament identification and interpretation of air photos, satellite images, geophysical maps, or the structural geology of the surface rocks. The statistical analysis of orientations of linear features is based on methods that have been extensively discussed elsewhere [e.g., Mardia, 1972; Abdel-Rahman and Hay, 1981].

Large-scale structures in the crust are expected to influence the locations of a variety of geological features that are nonlinear in form. For example, alkaline igneous complexes and kimberlites, ore bodies, and earthquakes could all be controlled by crustal structures with approximately linear traces (faults or fractures). This paper addresses these localized, pointlike features and their connection to linear features. Ideally, one would establish a one-to-one correlation between the locations of lineaments and pointlike features associated with them. In practice, this has rarely been achieved.

Furthermore, ambiguities arise even in such an apparently straightforward connection because the assumptions inherent in the interpretation of spatial correlations cannot be adequately tested. The relationship between "prominent" photolineaments and structural change points proposed by Werner [1979] illustrates the problem. Werner assumed that each lineament could be represented by a 2-km-wide zone and tested the spatial correlation between the zones and structural change points by comparing the number of points actually within the zones to the number expected from a random distribution of points. However, such a test does not address a fundamental uncertainty as to whether the "prominent" lineaments compose a set of observations that accurately represent the orientations of crustal structures.

Statistical tests applied to points and lineaments will only constrain hypotheses of geologic relationships between points and structures when lineaments correspond exactly to structures (Figure 1a). However, observed lineaments may underrepresent the structures that control the locations of the points (Figure 1b); this is likely when value judgments of the "prominence" of lineaments must be made. Alternatively, lineaments may overrepresent the structures if only a subset of the structures controls the locations of points (say, the largest structures) but all structures produce lineaments (Figure 1c). A final possibility is that the structures that control points and those that produce lineaments have no relationship (Figure 1d), including an extreme case in which there are structures but no lineaments. Tests of spatial correlation applied to cases in Figures 1b, 1c, or 1d have no clearly defined relevance to geological hypotheses, even if the test indicates a departure from randomness.

An alternative approach is developed in this paper. If structures with linear traces control the locations of pointlike features, then one type of evidence for such control and for the orientations of the structures can be obtained from the areal distribution of the points alone. The basis for such an analysis is not new: Chapman [1968] suggested that the locations of alkaline igneous complexes were related to crustal fractures and that the orientations of the fractures were apparent in reticulate or lattice patterns that could be recognized from the areal distribution of the complexes. He illustrated his lattices by drawing grid lines such that there was a "tendency for the plutons and central complexes to lie at nodal points in the chosen lattice..." [Chapman, 1968, p. 389]. A means for determining lattice directions other than by inspection was not proposed.

Paper number 5B5608.
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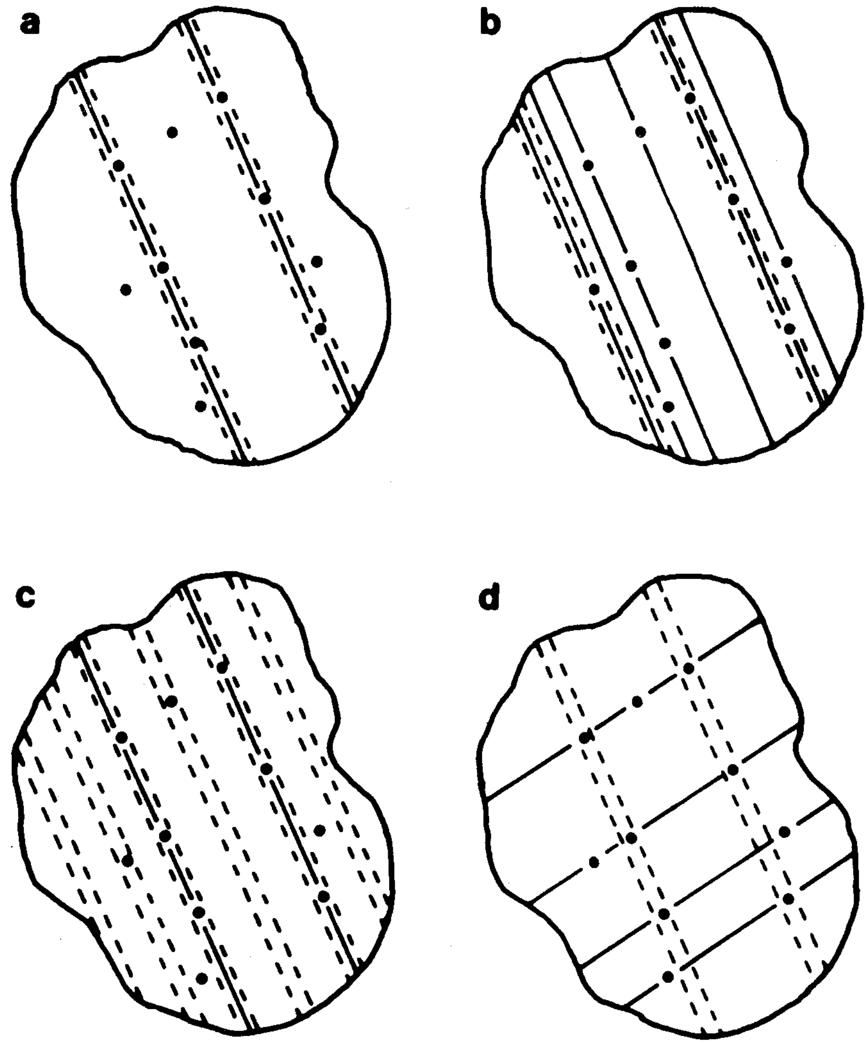

Fig. 1. Hypothetical relationships among pointlike features (dots), the structures that may control them (solid lines), and recognized lineaments (paired dashed lines). (a) Lineaments and structures coincide, some points not structurally controlled, (b) Lineaments underrepresent structures, (c) Lineaments overrepresent structures, (d) Lineaments do not represent structures. Spatial correlation of lineaments and pointlike features is relevant to structural control only in the situation in Figure 1a.

Fracture Traces as Lattices

By analogy with Chapman's proposal that crustal fractures selectively permit the emplacement of magma from depth, structures are here considered that behave as narrow zones in which the probability of an event (e.g., magma emplacement) is high relative to their surroundings. The traces of the structures on the surface of the earth describe an approximately planar lattice. A set of traces with a single orientation form a lattice of order 1; two sets with two different orientations define a lattice of order 2; and so on. The lattice controls the locations of pointlike features and creates a lattice distribution of points (Figure 2). The structural lattice imposes an anisotropy on the areal distribution of points.

The definition of a lattice used here differs from the more familiar concept of a crystal lattice. A crystal lattice is deterministic: given a unit cell, the locations of all ions in a larger crystal can be specified. The structural lattice is probabilistic: the distances between lattice lines and the locations of points along them are not specified; only the directions of the lattice lines are determined. In addition, the structural lattice may be sparsely populated compared to a crystal lattice. In terms of Chapman's proposal for the alkaline igneous complexes, only a small fraction of the nodal points may be occupied.

Thompson and Hager [1979] attempted to quantify the procedure of finding lattice directions by specifying the minimum number of points required to define a lattice line. The maxima in the frequency distribution of lattice line azimuths were proposed as directions of structural control. However, the criterion for selecting the minimum number of points is not explained.

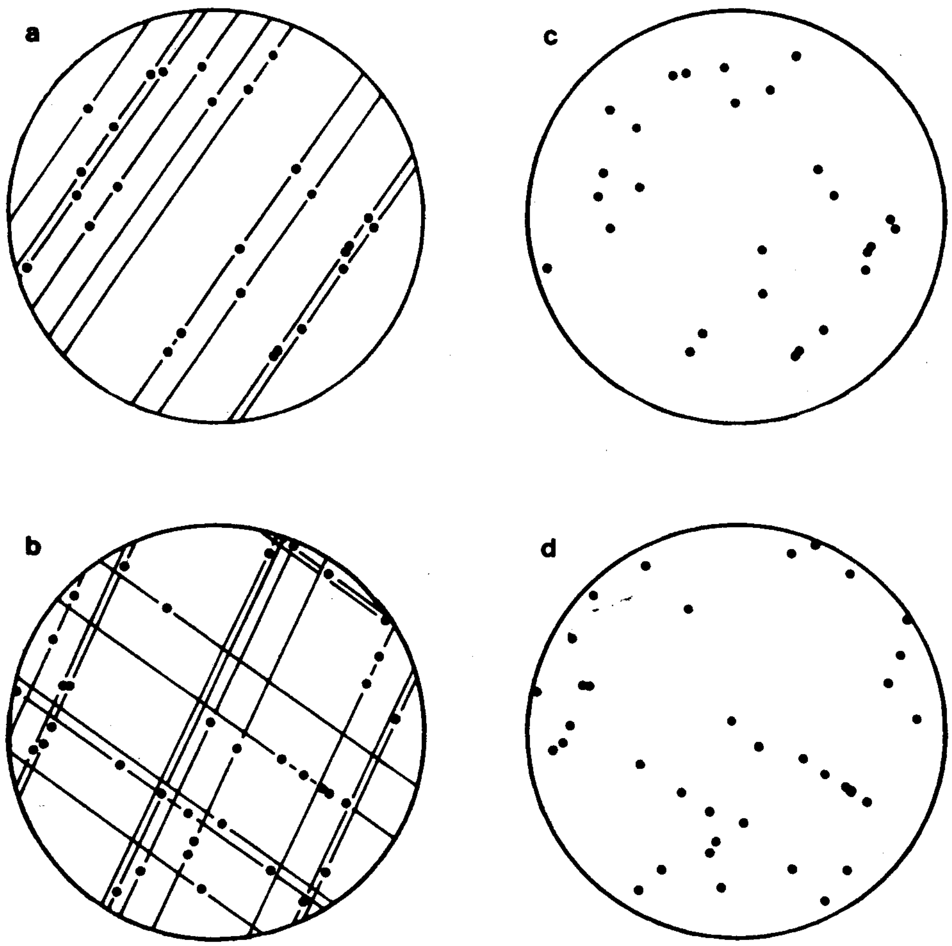

Fig. 2. Realizations of lattices and lattice distributions of points of order 1 (Figure 2a) and order 2 (Figure 2b) in circular regions. The order 1 lattice lines have an azimuth of 35°; the order 2 lines have azimuths of -55° and 25°. Azimuthal distributions of these realizations are shown in Figure 5. Figures 2c and 2d show the points of Figures 2a and 2b, respectively, without lattice lines.

A fundamental difficulty not recognized by Chapman [1968] or Thompson and Hager [1979] is the faulty intuitive perception of a lattice line as one that passes close to a large number of pointlike features relative to other choices of orientation. The number of points that fall arbitrarily close to a line depends on two different characteristics of areal distributions: shape and pattern. Shape refers to the areal measure of a distribution of points as defined by a closed curve containing them [Rogers, 1974]. Pattern refers to the intrinsic spatial ordering of points, for example, on a lattice. Shape may be independent of pattern in geological contexts. The shape of the outcrop area of a group of alkaline igneous complexes, for example, may depend on coincidental factors such as where erosion or deposition has revealed or covered the complexes or how an investigator chooses to define his study area. If the shape is elongated, lines parallel or subparallel to the direction of elongation may pass close to a large number of points regardless of any lattice pattern.

Most previous attempts to detect anisotropies in areal distributions of points have been based on criteria that are poorly defined or intuitive or that depend on arbitrary choices. Two-dimensional spectral analysis has proven useful in geography and forestry [Ripley, 1981], but spectral analysis is most effective when the anisotropic structure is periodic, a condition not expected for crustal structures. In order to quantify the search for lattice directions an appropriate statistic must be used to describe the areal distribution of points. The distribution of this statistic should provide a means to distinguish between areal distributions that are random and those that are anisotropic. The definition of the statistic and the procedure for comparing its distribution with that expected from random points must provide for removing biases caused by shape.

The standard criteria used to distinguish between random and nonrandom areal distributions are based on the distribution of nearest-neighbor distances [e.g., Kendall and Moran, 1963; Rogers, 1974; Bartlett, 1975]. However, a distance measure is not well suited to represent the directional ordering expected in lattice distributions. Furthermore, a lattice imposes long-range directional coherence that no nearest-neighbor

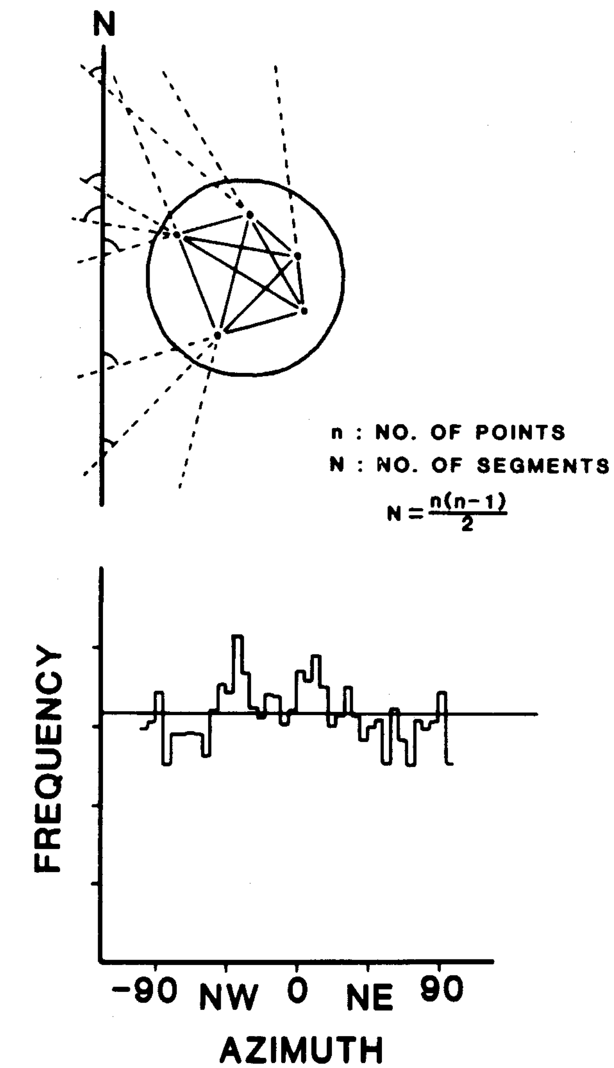

Fig. 3. Schematic representation of the construction of intersegment azimuths and the azimuthal distribution. Intersegments are constructed between all pairs of points; the angle of intersection of each segment or its extension with a N-S line defines its azimuth. The azimuthal distribution is a histogram showing the frequency with which azimuths occur. The horizontal line represents the average frequency per cell.

statistic could detect. The orientation of a line segment connecting any two points within the areal distribution as a statistic is proposed here (Figure 3). The azimuthal distribution of the statistic in the absence of shape effects is expected to be random if the areal distribution is random but modes are expected at the azimuths of any lattice directions (Figure 4). The details of the procedure are presented later.

Applications

Alkaline ring complexes are examples of discrete geological features that appear as singularities on a regional scale map. However, the application need not be restricted to discrete phenomena. Singularities defined by local maxima and minima in continuous field variables (gravitational, magnetic intensity), compositional variables (chemical concentration, mineralogic mode), or topographic elevation can also be analyzed. Analysis of maxima or minima in continuous data may be more reliable than interpreting lineaments in contoured data: extremae are first-order characteristics of data, whereas contouring involves a complex and somewhat arbitrary technique. Furthermore, the analysis could be extended to other approximately planar surfaces, for example, mineral distributions on thin sections.

In this paper, application of the procedure is illustrated with reference to the plutons of the White Mountain magma series [Chapman, 1968], structural change points in West Virginia [Werner, 1979], and magnetic anomalies in the Delaware-Pennsylvania Piedmont [Thompson and Hager, 1979].

Results

Azimuthal Distributions: Shape and Pattern

A line segment connecting two points in an areal distribution (intersegment) defines an azimuth. The azimuthal frequency distribution is sensitive to the presence of anisotropies: modes in the frequency distribution tend to occur in the lattice directions. This is illustrated for

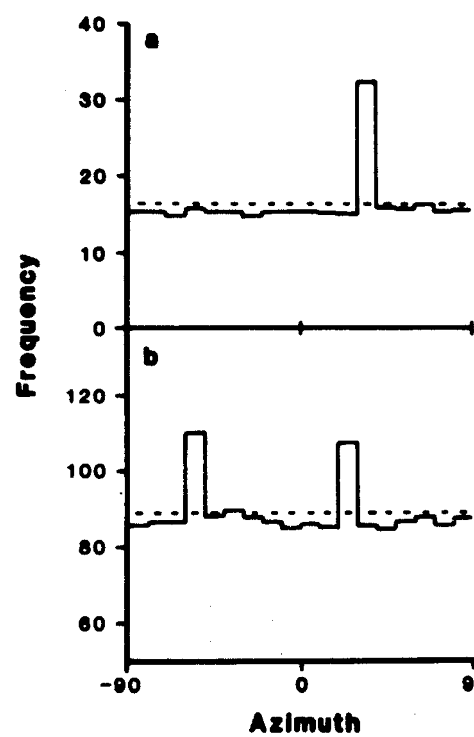

Fig. 4. Averaged azimuthal histograms from 100 random simulations of an order 1 lattice (Figure 4a) and an order 2 lattice (Figure 4b) in circular regions. The lattice directions and the expected distances between lines and between points on lines are identical to those used to generate the realizations in Figure 2. The dashed lines show the average frequency per cell.

lattices of order 1 (Figure 4a) and order 2 (Figure 4b) by average frequency distributions formed by Monte Carlo simulations of lattice distributions. In each case, the expected distances between lines and between points on lines, as well as the directions of the lines, were specified. However, the actual distances were determined by random exponential variates. Therefore each simulated lattice distribution is a result of two sequential Poissonian processes. The term "realization" will be used to refer to the result of carrying out a simulation based on expected values by means of random variates. The effect of shape on the frequency distributions has been eliminated by sampling a circular region of the lattice.

The modes of the average histograms accurately reflect the lattice directions (Figure 4). The sharpness of the peaks results from averaging many simulations: random variations tend to average out, while frequency in the lattice directions tends to accumulate. For any individual simulation (for example, those shown in Figure 2) much more variability in the size and location of the mode or modes is expected (Figure 5). Thus the azimuthal distribution can only be expected to detect anisotropies within probabilistic limits. For example, repeated simulations show that given the characteristics of the lattice of which Figure 2b is a realization, both of the anisotropic directions would be accurately represented by modes in one realization out of five. At least one direction would be determined accurately in seven cases out of 10. The efficiency of detecting a lattice from the azimuthal distribution is improved if more points are included.

Figures 4 and 5 show how the intersegment azimuthal distribution responds to a lattice pattern in the absence of shape effects. Similarly, the effect of shape in the absence of pattern is considered: Average frequency distributions are generated from simulations of random patterns of points within curves bounding elongate regions, for example, ellipses (Figure 6). Once again, the averaging of large numbers of simulations reduces random variation. A mode occurs in the direction of maximum elongation, and the height of the mode is roughly proportional to the square of the elongation ratio (major axis/minor axis). The distribution quantifies the strong bias involved in attempts to "see" lattice directions. The same bias affects any method based on finding lines that pass close to the largest number of points.

When both a lattice pattern and an elongation are present in an areal distribution, the frequency modes that can result independently from pattern and shape are superimposed. If the analysis were restricted to circular regions the shape effect could be eliminated. However, depending on the density of points and the shape, too few points may occur within any circular region. Furthermore, artificially restricting the shape prevents the investigator from selecting a study area based on geologic criteria. An alternative solution developed here is to separate the effects of shape and pattern. Separation is possible because the shape effect is characterized completely by the configuration of a curve bounding the areal distribution. Therefore it is possible to use Monte Carlo simulations to find the azimuthal frequency distribution that results from shape alone and use it as a null distribution against which the observed distribution can be compared.

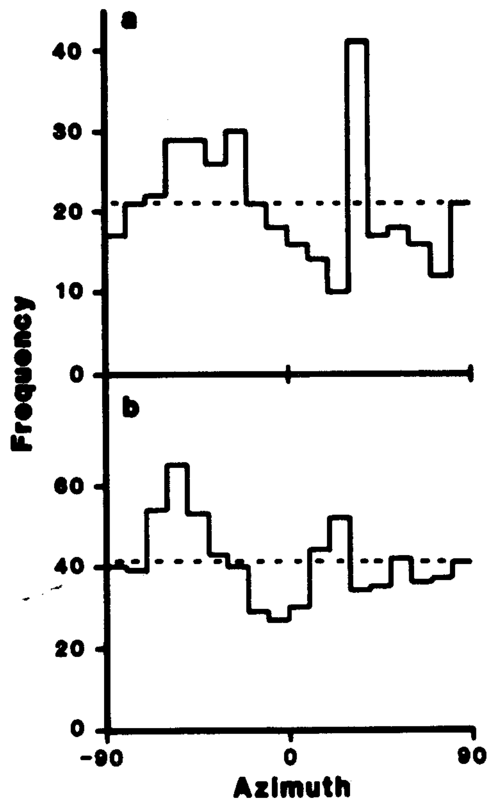

Fig. 5. Azimuthal histograms of the lattice realizations shown in Figure 2. Figure 5a results from the distribution in Figure 2c; and Figure 5b results from the distribution in Figure 2d. The dashed lines show the average frequency per cell.

The proposed procedure is shown schematically in Figure 7. An areal distribution of n points is characterized by a shape curve; a bounding polygon is used in this paper (Figure 7a). Repeated simulation of n random points within the shape curve can be used to form the distribution of frequency within each histogram cell which occurs from random variations in the pattern of points (Figure 7b). The mean of the simulated frequency in each cell can be used to normalize the histogram derived from the observed points, consequently eliminating the shape effect. The limit of random frequency variation at some level (say 95%) can be used to establish the likelihood that modes in the normalized histogram exceed the frequency expected from random variation (Figure 7c). The directions indicated by modes are the most probable directions of anisotropy.

Construction of Azimuthal Histograms

The histograms of intersegment azimuths are not meaningful unless the effects of uncertainties in the locations of points are included. The locations of pointlike features must be estimated: For example, the center of an alkaline igneous complex is estimated from the map distri-

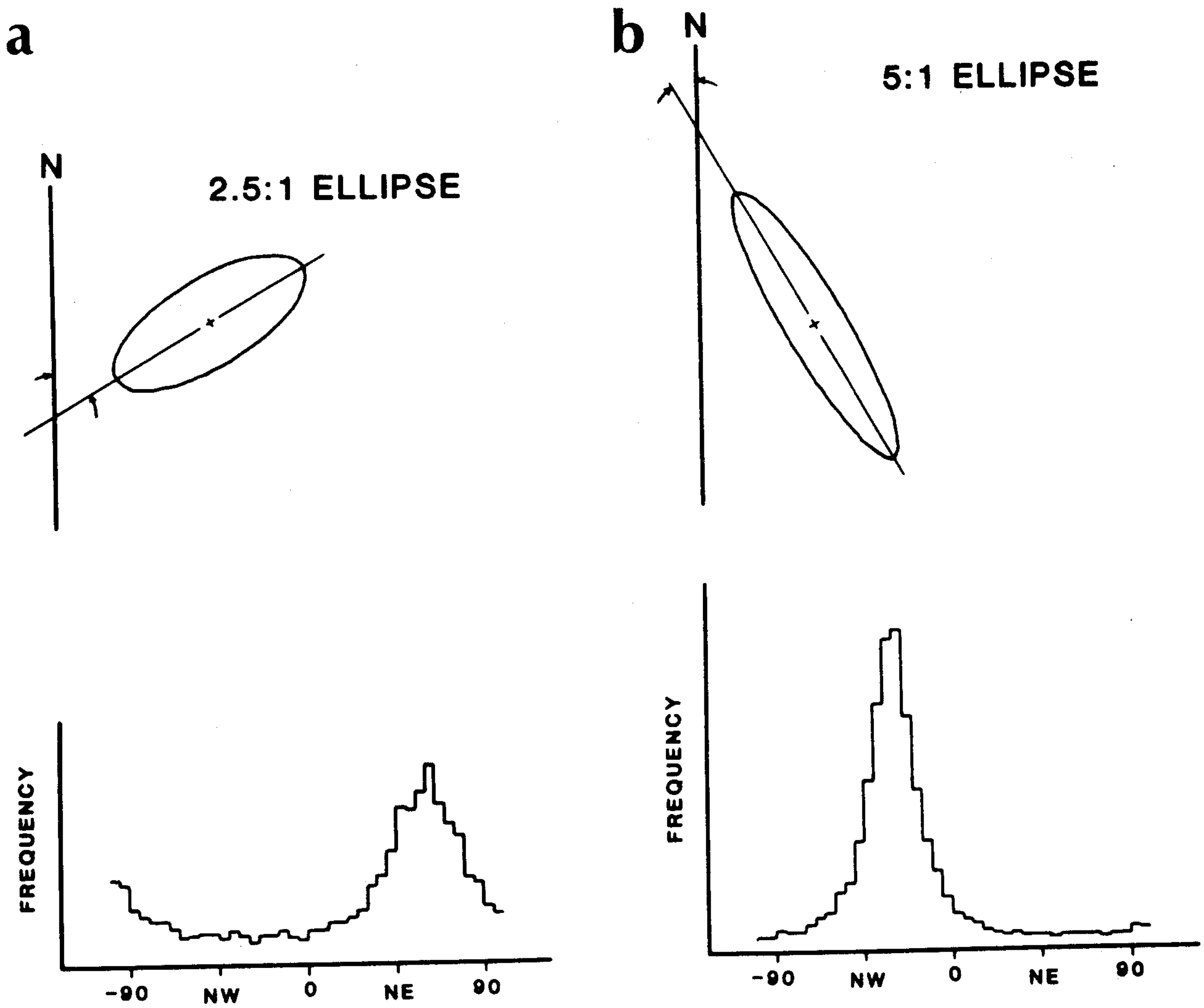

Fig. 6. Average azimuthal distributions resulting from random distributions of points in elongated regions. Azimuthal distributions were calculated for 50 points generated at random within the shape curves shown. The distributions shown are the averages of 100 sets of such simulations. The shape curves are (a) an ellipse with an elongation ratio of 2.5:1 with its major axis oriented N60°E and (b) an ellipse with an elongation of 5:1 with its major axis oriented N30°W.

bution of alkaline igneous rocks which form the complex. The size of the outcrop area and irregularities in its form, or in exposure, lead to uncertainty in its location. The distance between two points also must be taken into account: the farther apart they are, the smaller the contribution of uncertainties in their locations to the uncertainty of their intersegment azimuth.

The azimuthal uncertainty can be modeled as a statistical distribution that describes the probability that the azimuth falls within certain limits (Figure 8). Provided that the angular uncertainty is small compared to 180°, the univariate normal distribution is sufficiently accurate. Azimuths that have large angular uncertainties contribute negligibly to the formation of modes and can be deleted from the analysis.

The frequency distribution of azimuths is formed by summing the probabilities of the individual intersegment azimuthal distributions over the histogram cells (Figure 8c). The histogram contains a net frequency equal to the number of intersegments, N. N is related to the number of points, n, by $N = n(n-1)/2$. If there are k histogram cells, the expected frequency in each cell, $\tilde{c} = n(n-1)/2k$. For $k=18$, if $n = 20$, $\tilde{c} = 10.6$; if $n = 50$, $\tilde{c} = 68.1$; and if $n = 100$, $\tilde{c} = 275$.

To compare the observed histogram with the simulated histogram, the uncertainties in the location data must be incorporated in the simulation. It is not clear how this can be done most effectively. In this paper the uncertainties of the observed points are assigned randomly to the simulated points. This straightforward choice assumes that the location uncertainties can be treated independently of the locations. In other words, the lattice-forming process is assumed to affect the arrangement of point features without any effect on the degree with which we can locate them. The veracity of this assumption must be considered with regard to individual applications.

Statistical Analysis

The analysis of the azimuthal distribution of intersegments seeks to determine whether an observed distribution is consistent with a random pattern of points. More specifically, one tests whether or not the frequency within a given histogram cell is the result of a random pattern. Two statistical hypotheses can be formulated: (1) a null hypothesis, H_o, in which the cell frequency is that expected from random points, and (2) an alternative hypothesis, H_a, in which the cell frequency exceeds that expected from random points. Cell frequencies significantly less than expected from random points are not predicted by the lattice model and are not considered as part of the alternative hypothesis. The hypotheses pertain individually to each cell which forms the histogram; therefore a test on any particular cell is independent of the other cells.

A fiducial distribution consistent with the null hypothesis must be available to compare to the observed distribution. In applications to orientations of linear features the binomial distribution (with equal probability in all cells) is often applied [e.g., Hay and Abdel-Rahman, 1974]. However, while the azimuth of a random line has an equal chance of falling within any azimuthal cell, an azimuth defined by two points chosen at random may not because of the shape effect. Although the normalization of the observed distribution eliminates the shape effect, it obviously does so by transforming the cell frequencies, and the applicability of the binomial distribution to the transformed data is not certain.

An alternative to the binomial distribution is an empirical distribution based on an ensemble of Monte Carlo simulations, each simulation based on n random points located within the bounding polygon. The empirical distribution of frequencies within a particular azimuth cell i (Figure 7b) provides a fiducial distribution if a large number of simulations are made. The comparison between observed and fiducial distributions is thus made individually for each of the defined histogram cells. This approach contrasts with the use of the binomial distribution and eliminates a problem: histograms must be tested for departure from uniformity with regard to specific alternative hypotheses (e.g., unimodal, bimodal, etc.). Since the number of modes is not known a priori, general statements about significance levels cannot be made. However, the empirical distributions pertain to individual cells, and thus hypotheses about cell frequency can be tested without reference to the form of the entire azimuthal distribution.

The normalized azimuthal cell frequency and the limiting frequency at some significance level α can be calculated on the basis of the expected cell frequency, $\tilde{c}$, and the mean frequency of cell i for the empirical distribution, $\tilde{e}_i$ ($i = 1,k$). The normalized cell frequency $\hat{e}_i$ is defined as

$$\hat{e}_i = (\tilde{c}/\tilde{e}_i)\cdot e^*_i$$

where e^*_i is the observed frequency of cell i. Similarly, a critical value of the empirical distribution, $L_i(\alpha)$, is related to the critical

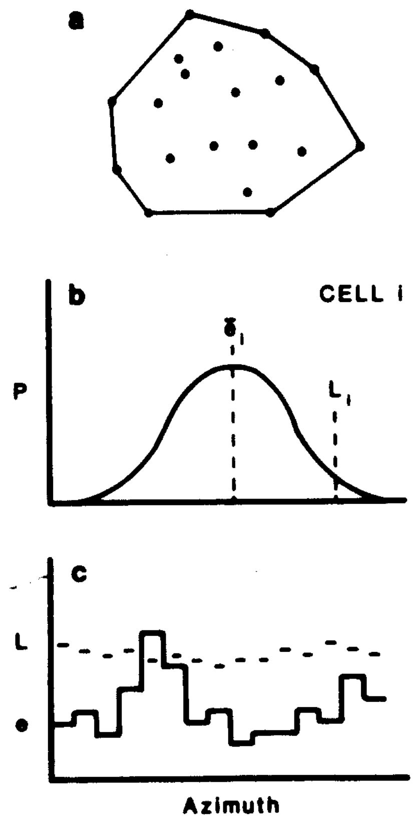

Fig. 7. A schematic illustration of some aspects of the azimuthal analysis. (a) A convex bounding polygon characterizes the shape of the areal distribution of observed points. (b) Monte Carlo simulations of random points within the bounding polygon are used to generate empirical probability distribution functions (P) of the frequency within individual histogram cells. The mean frequency of cell i, $\tilde{e}_i$, is a measure of the shape effect in the absence of anisotropy and is used to correct the observed frequency of cell i. A value of frequency, L_i, which is exceeded with probability α (the area under the curve to right of L_i is α), is the empirical upper critical value at the α significance level. (c) The corrected cell frequencies and the critical values for each cell are used to construct the corrected azimuthal distribution. The azimuths of cells with frequencies higher than the critical values (dashed line) are directions of anisotropy at the α significance level.

value for the normalized distribution by

$$\hat{L}_i(\alpha) = (\tilde{c}/\tilde{e}_i)\cdot L_i(\alpha)$$

Provided that the empirical frequency in a cell is sufficiently large the distribution is approximately normal and the critical points can be estimated from the standard deviation s_i, using the t distribution:

$$\hat{L}_i(\alpha) = (\tilde{c}/\tilde{e}_i)\cdot t(\alpha,v)\cdot s_i$$

where v is one less than the number of simulations used to generate the empirical distribution.

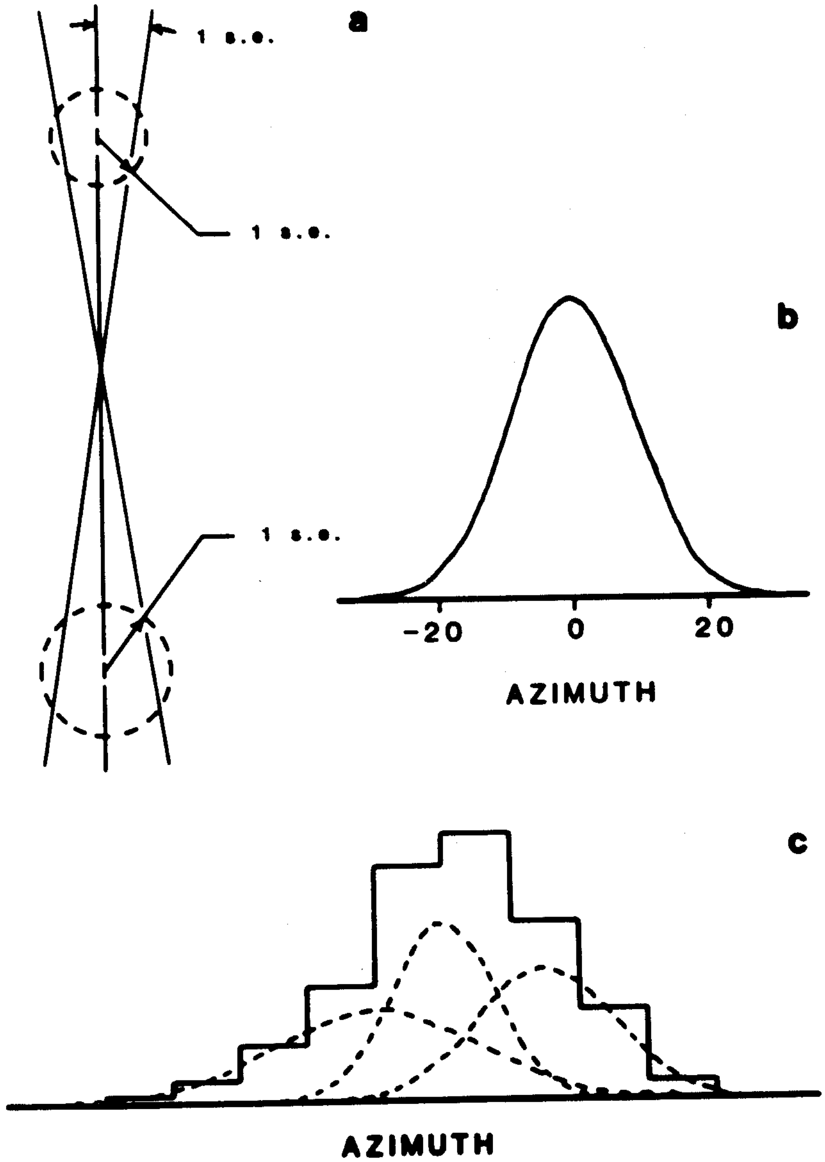

Fig. 8. A schematic illustration of the formation of observed azimuthal distributions from points whose locations are subject to uncertainty. (a) Location uncertainties (dashed circles with radii of one standard error) lead to an angular uncertainty on the estimate of the azimuth (solid lines diverging from center line). (b) The angular uncertainty on a single intersegment azimuth is represented by a probability distribution function; the highest probability is in the direction of the segment connecting the points. (c) The probability distribution functions for each intersegment azimuth are integrated within histogram cells to form the azimuthal distribution. Each azimuth may contribute to the frequency in more than one cell.

Corrected frequencies $\hat{e}_i$ that exceed the critical value $\hat{L}_i(\alpha)$ have a probability of α or less of resulting from an isotropic distribution of random points. For small α, say 0.05, it is highly probable that an anisotropy exists in the direction represented by cell i. However, as noted previously, even areal distributions generated by Monte Carlo simulations of a lattice process do not always yield azimuthal histograms with modes that accurately indicate the anisotropy. If $\hat{e}$ is less than $L(\alpha)$ for each cell, there is no clear evidence for anisotropy: either no anisotropy exists, or the anisotropy cannot be detected from the data.

Whether the calculated critical value actually rejects H_o at the α significance level depends on how well the simulations reproduce the statistics of the observed distribution in every way other than with respect to anisotropy. For example, if the points within the bounding polygon actually composed two subregions with different densities of points, simulations of a single homogeneous distribution would not be comparable. However, the effect of heterogeneous densities on the correlation process and significance levels is difficult to predict. As a first approximation, visual inspection of data to avoid obvious heterogeneities should be made. More sophisticated means to detect and characterize density variations [e.g., Diggle, 1983] can be utilized in future work. The analysis of point patterns is so complex that the critical values should be treated more as suggestions of significance rather than as hard and fast limits, as in the manner that Mosteller and Tukey [1977] refer to indicators. Cells that approach or exceed a chosen limit should invite further statistical study or field work rather than outright rejection or acceptance of the null hypothesis.

Theoretically, there is no limit to the number of points which can be used in the analysis. As noted previously, the efficiency of detecting lattice distributions is diminished by small numbers of points. However, the efficiency also depends on several other factors not known a priori and thus guidelines for a lower limit to n would not be meaningful. However, a practical upper limit of several hundred points exists, since the time required to perform the calculations increases roughly as n^2.

Applications

White Mountain magma series. Chapman [1968] recognized "strikingly apparent" NNW and E-W trending lattice directions in the plutons of the White Mountain magma series. To test Chapman's hypothesis, azimuthal histograms for the locations of igneous complexes in New Hampshire and Maine were constructed. The locations were determined from state geologic maps (New Hampshire [Billings, 1955] and Maine [Hussey, 1967]) and include virtually all complexes shown by Chapman [1968, Figure 29-2]. Location uncertainties for the center of each complex were based on the size of the complex and the degree to which a center was defined by ring structure or by its shape. The locations of 34 points included in the analysis are shown in Figure 9. Some complexes shown on Chapman's map were not used in this study because they fell in the fringes of the main group of complexes and tended to form heterogeneous regions within the shape curve.

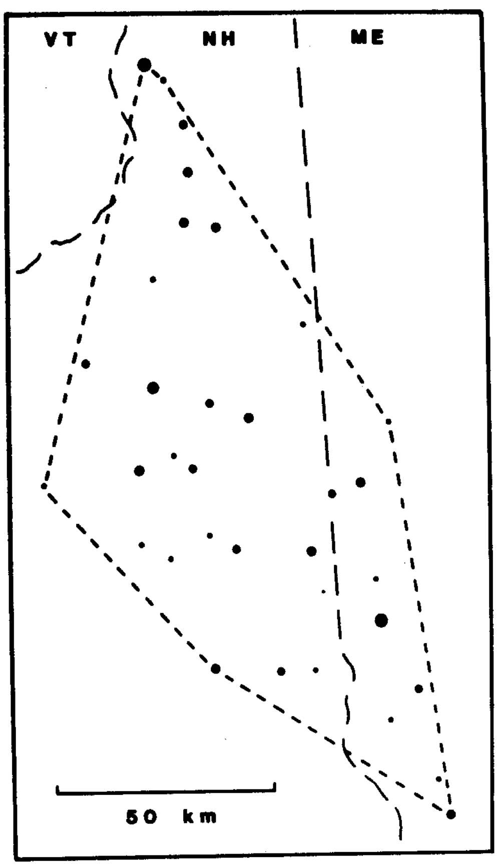

Fig. 9. Map of 34 White Mountains complexes used in the azimuthal analysis (Figure 10). The radius of each point is equal to the uncertainty on the location. The short-dashed lines show the bounding polygon defined by six of the points.

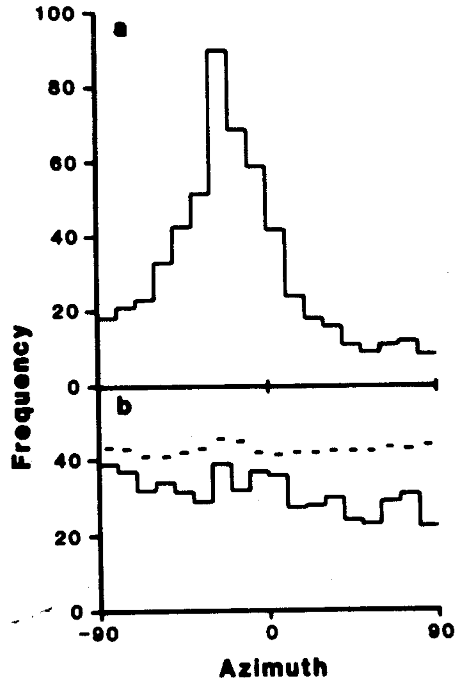

Fig. 10. Azimuthal distributions for 34 plutons of the White Mountain magma series (Figure 9). The uncorrected distribution (Figure 10a) contains a large, broad mode in the NNW direction of elongation. This mode is absent in the corrected distribution (Figure 10b), and no cell frequencies approach the 95% critical values (dashed line). The average frequency per 10° cell is 31.2.

The 34 complexes define 561 intersegments, each of which has a well-defined azimuth when the uncertainties are taken into account. The azimuthal frequency distribution of the data is shown in Figure 10a. The shape of the area in which the complexes crop out is defined by a convex polygon (Figure 9). Thirty-four points generated at random within the polygon and assigned the uncertainties estimated for the observations yield a distribution of azimuths for a realization of a random pattern. Any modes in the distribution are expected to result solely from the shape effect. However, any single realization of a random pattern does not provide a good estimate of the shape effect. Therefore 300 separate simulations of 34 points each are used to define empirical probability distribution

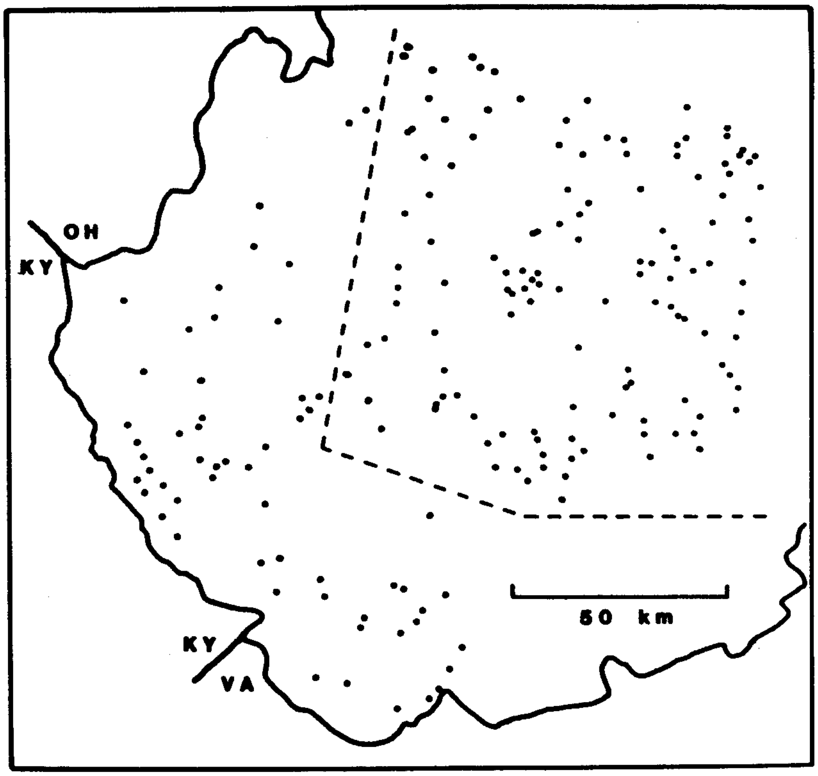

Fig. 11. Map of structural change points in West Virginia [from Werner, 1979, Figure 4]. The size of the points is approximately equal to the uncertainty in their locations. The dashed line delimits a group of 128 points that were used in the azimuthal analysis (Figure 12).

functions (Figure 7b) and to calculate a corrected azimuthal distribution for the White Mountains (Figure 10b).

The complexes define a NNW trending outcrop area. The effect of the elongation on the azimuthal distribution is obvious by comparison of the uncorrected distribution (Figure 10a) with the corrected distribution (Figure 10b). The corrected distribution contains no modes that approach the 95% critical value. The striking arrangement perceived by Chapman (NNW and E-W grid lines) was evidently a result of the shape of the White Mountain province and not the result of an intrinsic lattice pattern. It should be emphasized that the azimuthal analysis does not prove that linear crustal structures have not controlled the locations of the White Mountain plutons but that such an hypothesis cannot be proven using the available data.

West Virginia: Structural change points. Werner [1979] suggested that structural change points (SCP) in southern West Virginia were concentrated along photolineaments representing the 38th parallel lineament. The distribution of the SCP on Werner's [1979, Figure 4] map suggest that their density may be somewhat lower in the southwestern portion of the state. A homogeneously distributed subgroup (roughly east of 81°45'W longitude) of 128 SCP (Figure 11) were used to construct an azimuthal histogram. Their locations were estimated from Werner's figure. The uncertainty in this case was dominated by measurement precision because of the small scale of the map. The standard error in x and y co-

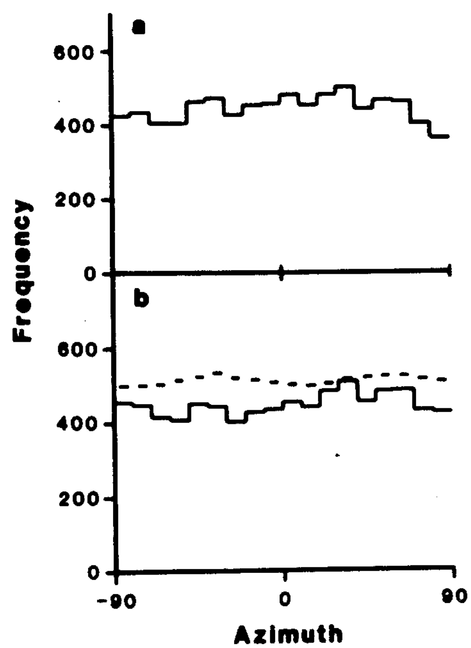

Fig. 12. Azimuthal distributions for 128 structural change points in southwestern West Virginia [Werner, 1979] (see details in text). The uncorrected distribution (Figure 12a) and the corrected distribution (Figure 12b) are similar: the area is not highly elongated (Figure 11). The N35°E cell nearly equals the 95% critical value. The average frequency per 10° cell is 449.

ordinates of each point was assigned to be 1 km. However, the average intersegment distance is over 60 km and the effect of the uncertainty is probably small.

The shape of the study area is not strongly elongated as reflected by the similarity between the uncorrected and corrected azimuthal histograms (Figure 12). A near approach to the 95% critical value occurs in the cell centered on N35°E. This direction does not correspond well with the orientations of any of the sets of photolineaments in this part of West Virginia [Werner, 1979, Figure 2] and is substantially different from the trend of the 38th parallel lineament (N65°E). Magnetic axes [Werner, 1979, Figure 6] trend N35°E in the northern part of the study area, and a connection between the features that cause these anomalies and the SCP seems more likely than a connection between the SCP and photolineaments.

Delaware-Pennsylvania piedmont: Magnetic closures. Thompson and Hager [1979] proposed that several nearly orthogonal sets of crustal fractures could explain the orientations of faults, joints, and magnetic contour lineaments in the rocks that make up the piedmont of Delaware and southeastern Pennsylvania. They also suggested that preferred directions in the distribution of magnetic closures reflected a similarly complex distribution of fractures oriented: N85°W and N5°E, N50°W and N35°E, and N35°W and N48°E [Thompson and Hager, 1979, Figure 8]. They suggested that magnetic highs and magnetic lows individually followed the same patterns.

To test their hypothesis, the locations of magnetic contour closures were measured from Thompson and Hager [1979, Figure 9]; a total of 125 closures was found (Figure 13). The uncertainty on each point was made proportional to the average size of the closure. Individual azimuthal analyses of 53 magnetic highs and 72 magnetic lows confirm the hypothesis that they follow the same pattern. However, the pattern is more simple than Thompson and Hager hypothesized. The lows define a mode extending from N60°E to N80°E, and the highs define a mode extending from N40°E to N70°E: both modes are broad and overlap. Analysis of the combined data yields a broad mode centered on N60°E: four adjacent 10° histogram cells exceed the 95% critical value (Figure 14). There is no suggestion of northwesterly trending anisotropies or orthogonal anisotropies. The width of the mode may indicate that several sets of structures exist with nearly parallel traces or that the structures are not linear but arcuate.

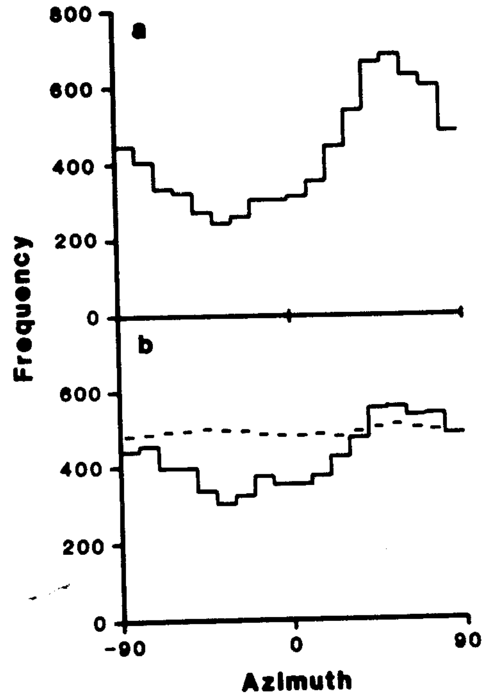

Fig. 14. Azimuthal distributions for 125 magnetic closures in the piedmont of Delaware, Pennsylvania, and Maryland [Thompson and Hager, 1979] (see text for details). The uncorrected distribution (Figure 14a) contains a large, broad mode in the ENE direction of elongation. However, this mode is only partly due to shape since it persists in the corrected distribution (Figure 14b). Four adjacent 10° cells centered on N60°E exceed the expectations of a random pattern at the 0.05 significance level. The average frequency per cell is 428.

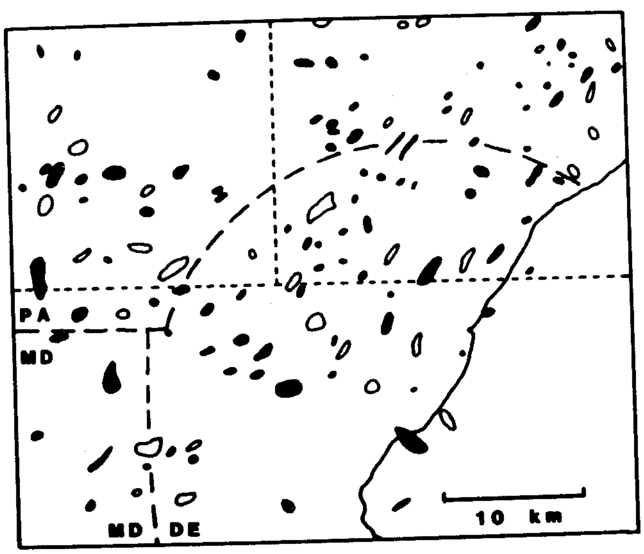

Fig. 13. Map of magnetic anomaly contour closures [from Thompson and Hager, 1979, Figure 9]. Magnetic highs are shown by solid shapes; magnetic lows by open curves. The entire set of 125 closures was used in the analysis shown in Figure 14. The dashed lines delimit two subregions that were analyzed separately (Figure 15): a set of 41 points south of the E-W line and a set of 60 points to the east of the N-S line.

Discussion

Two aspects of azimuthal analysis that require further discussion are the method of locating various features by means of their centers and the uncertainties that are attached to these locations. Two cases can be distinguished with regard to locating centers:

1. Mapped features are those that are of

measurable size on the scale of interest and which therefore have shapes. For example, igneous complexes may differ in their degree of completeness, but it may be possible to approximate the shape or an estimated reconstruction of the shape by simple geometric forms such as circles, ellipses, or combinations of them. The geometric center of such an ideal shape can then be constructed.

2. Defined features are those that are essentially "dimensionless" and for which a shape curve would not be appropriate. Examples might be earthquake epicenters or structural change points. For these features the location of the center may be given numerically in a primary reference or may be represented by a "point" on a map.

Locations can be measured in virtually any coordinate system provided that it is oriented properly with respect to north. However, care must be taken to avoid introducing distortions that affect angular relationships. For example, xerographic copying of maps can lead to anisotropic distortion of scale and should be avoided.

Uncertainties on locations include both precision and accuracy. Precision is relatively easy to characterize since it depends on the reproducibility of measurements. Important factors may be the scale of the map, the fineness of divisions on a measuring rule, or the characteristics of a digitizing table. However, precision is probably an insignificant portion of the total uncertainty in the case of mapped features.

Accuracy relates to the "meaningfulness" of a location in terms of a hypothetical model for the origin of the feature. For example, an igneous complex may be partially exposed in the field as a roughly semicircular mass; other complexes in the same province that are well exposed may be central complexes with nearly circular forms. The most straightforward assumption is that the former, too, is circular in form. However, the accuracy of this assertion may be nearly impossible to evaluate.

In many cases of practical interest the median distance between points is much larger than any realistic uncertainty. In such a case, the uncertainties contribute little to the standard errors of most of the individual azimuths (Figure 8) and therefore have little effect on the azimuthal distribution. Likewise, an overestimate or underestimate of the uncertainties only affects the relatively small proportion of azimuths for which the interdistance is small compared to the uncertainty. However, if the median distance is comparable to the size of the uncertainties, misestimates affect a large proportion of the azimuths and may be more important to the azimuthal distribution.

The uncertainties on the observations are reproduced by the Monte Carlo simulations. Therefore it is unlikely that misestimates of uncertainties could lead to a false indication of anisotropy. If false modes are to be avoided it may be better to overestimate rather than underestimate uncertainties.

Subsampling Areal Distributions

The interpretation of azimuthal distributions rests on the assumption that one and only one lattice pattern exists everywhere in the study area. Subsampling an areal distribution can be used to test the validity of this assumption and to characterize departures from it. A homogeneous lattice distribution should yield the same azimuthal distribution for data taken from any subregion. If the distribution is heterogeneous, then different directions of anisotropy may be found in different subregions. Subsamples could yield anisotropies even if a larger region does not: regional variations in lattice patterns may effectively mask the pattern on the large scale.

To illustrate how subsampling works, consider the magnetic contour closure data from Thompson and Hager [1979]. The azimuthal distribution for their data has a broad ENE mode that suggests that there are either several subparallel sets of structures or that the structures are arcuate rather than linear. The latter hypothesis receives support from the fact that magnetic contour lineations trend E-W in the southern part of the mapped region [Thompson and Hager, 1979, Figure 4]. As a test, two subregions were analyzed: a set of 41 points south of approximately 39°45'N; and a set of 60 points north of 39°45'N and east of 76°40'W (Figure 13). The azimuthal distributions are similar to one another (Figure 15) and to the distribution for entire data set (Figure 14). These results thus support the hypothesis of subparallel features rather than regional variations in direction.

Data could also be sorted in ways other than by area. For example, sorting magnetic anomaly data by type (high or low) has already been covered in the results section. Magnetic anomalies could also be sorted with respect to the basement lithology with which they are associated, although there are too few data in Thompson and Hager's study to attempt this. Igneous complexes within a single province may have been emplaced at different times and could be sorted by age.

Subsampling can also help to interpret results that might otherwise be ambiguous. For example, an azimuthal mode that does not exceed the 95% critical value could result from either an essentially random pattern or from a pattern with a weak anisotropy. Data from different subareas of a random pattern are expected to yield independent azimuthal distributions: there is an equal probability of a mode occurring in any histogram cell. However, a pattern with a weak anisotropy might produce similar azimuthal distributions from different subsamples. Thus subsamples that produce modes in the same direction are likely to come from an anisotropic pattern although the mode might not exceed the 95% value in any single case.

A Problem: Structural Grids

Azimuthal analysis determines the trends of structures but does not provide any other information about their spatial distribution. For example, commonly utilized statistics pertaining to fractures such as average spacing or average length cannot be obtained. The analysis does not locate individual structures. Thus it does not provide a straightforward alternative to spatial correlation of pointlike features and structures.

The inherently probabilistic nature of azi-

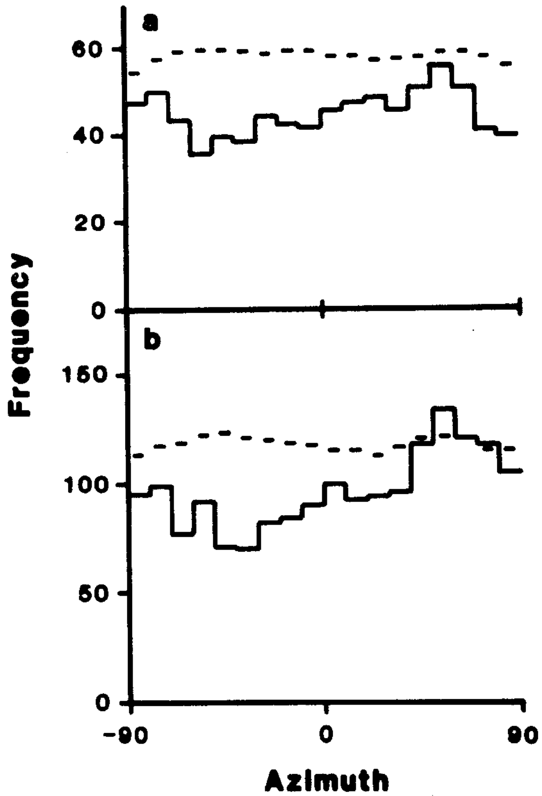

Fig. 15. Comparison of corrected azimuthal distributions of magnetic contour closures in the Piedmont [Thompson and Hager, 1979] (see text for details) from two different subareas. The patterns of both the southern subarea (Figure 15a) and the northeastern subarea (Figure 15b) are similar to that of the entire area (Figure 14b). A homogeneous pattern of subparallel, ENE trending anisotropies is suggested.

muthal analysis guarantees that any spatial description of the structures will also be probabilistic. Thus structural grids of the type proposed by Chapman which associate every point with a line are unrealistic. An alternative to a deterministic grid is one that shows the most likely locations of those structures that are associated with a relatively large number of points. The construction of even such a provisional grid would be complex and will not be considered in this paper.

Conclusions

The large-scale structures of the earth's crust are expressed in a wide variety of forms: no single type of data could be expected to provide a complete description of them. Similarly, no single type of analysis can be entirely adequate. The information derived from azimuthal analysis may be complementary to that given by standard spatial statistics, and the information provided by pointlike features may be complementary to that derived from linear features.

The importance of azimuthal analysis is that it yields new information from a previously untapped, or poorly tapped, set of data. Other methods for dealing with directional characteristics of pointlike features have been discussed in the introduction; they fail in one of two ways:

1. They do not extract directional information intrinsic to areal distributions of points but instead compare the locations of points to the locations of linear features, the orientations of which must be specified prior to the analysis,
2. They rely on arbitrary or inaccurate concepts of how intrinsic patterns can be identified.

Azimuthal analysis is an improvement on other methods because of three innovations:

1. The azimuth of an intersegment is used as the fundamental measure of an areal distribution of points. The frequency distribution of intersegment azimuths is constructed from all pairs of points; therefore it is sensitive to both long- and short-range patterns expected from lattice distributions. Without such a measure it would not be possible to quantify anisotropy.
2. The azimuthal distribution responds to both pattern and shape. The shape of an areal distribution is characterized by a bounding polygon that makes it possible to quantify the shape effect via Monte Carlo simulations of random patterns. Thus the observed azimuthal distribution can be corrected for shape. Without a shape correction an analysis of anisotropy in elongated areas would be inaccurate.
3. Monte Carlo simulations provide empirical confidence values for azimuthal frequencies that can be used as guides to interpreting azimuthal distributions. Distributions that exceed or approach specified limits invite further work, either by statistical means or by gathering more data in the field.

The use of azimuthal analysis does not guarantee that all structural anisotropies within a given area can be detected. Rather, it provides a quantifiable, reproducible limit to the directional information that can be reliably extracted from a point pattern. Applications of azimuthal analysis to data from other published reports suggest that there has been a tendency to overestimate the complexity of information available from point patterns.

Acknowledgments. The manuscript benefited from reviews by D. P. Adam, Alan Gillespie, and an associate editor of JGR. Lucian Platt, Reginald Shagam, Mary Emma Wagner, Donald Wise, Colin Howard, LeeAnn Srogi, and Arnie Sugar provided criticism of an earlier version of the manuscript. Computing was funded by the University of Pennsylvania.

References

Abdel-Rahman, M.A., and A.M. Hay, Sampling and statistical analysis of multi-modal orientation data, in Proceedings of the Third International Conference on Basement Tectonics, edited by D.W. O'Leary and J.L. Earle, pp. 73-86, Basement Tectonics Committee, Denver, Colo., 1981.

Bartlett, M.S., The Statistical Analysis of Spatial Pattern, 90 pp., Chapman and Hall, London, 1975.

Billings, M.P. (Compiler), Geologic map of New

Hampshire, scale 1:250,000, U.S. Geol. Surv., Reston, Va., 1955.

Chapman, C.A., A comparison of the Maine coastal plutons and the magmatic central complexes of New Hampshire, in Studies of Appalachian Geology, Northern and Maritime, edited by E. Zen et al., pp. 385-396, John Wiley, New York, 1968.

Diggle, P.J., Statistical Analysis of Spatial Point Patterns, 148 pp., Academic, Orlando, Fla., 1983.

Hay, A.M., and M.A. Abdel-Rahman, Use of chi-square for the identification of peaks in orientation data: Comment, Geol. Soc. Am. Bull., 85, 1963-1966, 1974.

Hussey, A.M. (Chief Compiler), Preliminary geologic map of Maine, scale 1:500,000, Maine Geol. Surv., Augusta, Maine, 1967.

Kendall, M.G., and P.A.P. Moran, Geometrical Probability, Griffin, London, 1963.

Mardia, K.V., Statistics of Directional Data, 357 pp., Academic, Orlando, Fla., 1972.

Mosteller, F., and J.W. Tukey, Data Analysis and Regression: A Second Course in Statistics, 588 pp., Addison-Wesley, Reading, Mass., 1977.

Ripley, B.D., Spatial Statistics, 252 pp., John Wiley, New York, 1981.

Rogers, A., Statistical Analysis of Spatial Dispersion, 164 pp., Pion, London, 1974.

Thompson, A.M., and G.M. Hager, Lineament studies in structural interpretation of a stabilized orogenic region: Appalachian Piedmont, Delaware and adjacent Pennsylvania, in Proceedings of the Second International Conference on Basement Tectonics, edited by M.H. Podwysocki and J.L. Earle, pp. 74-85, Basement Tectonics Committee, Denver, Colo., 1979.

Werner, E., The 38th parallel lineament in West Virginia, in Proceedings of the Second International Conference on Basement Tectonics, edited by M.H. Podwysocki and J.L. Earle, pp. 406-412, Basement Tectonics Committee, Denver, Colo., 1979.

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(Received April 22, 1985;
revised July 15, 1985;
accepted July 30, 1985.)