8 DRAINAGE BASIN ANALYSIS

INTRODUCTION

Horton devised a quantitative method of analyzing drainage basins which has become a standard technique for presenting data on drainage basins. It is based upon a hierarchy of stream ordering which was revised by Strahler as follows: the fingertip tributaries are first order streams, two first orders combine to form a second order, two second orders form a third, etc. (Fig. 8.1).

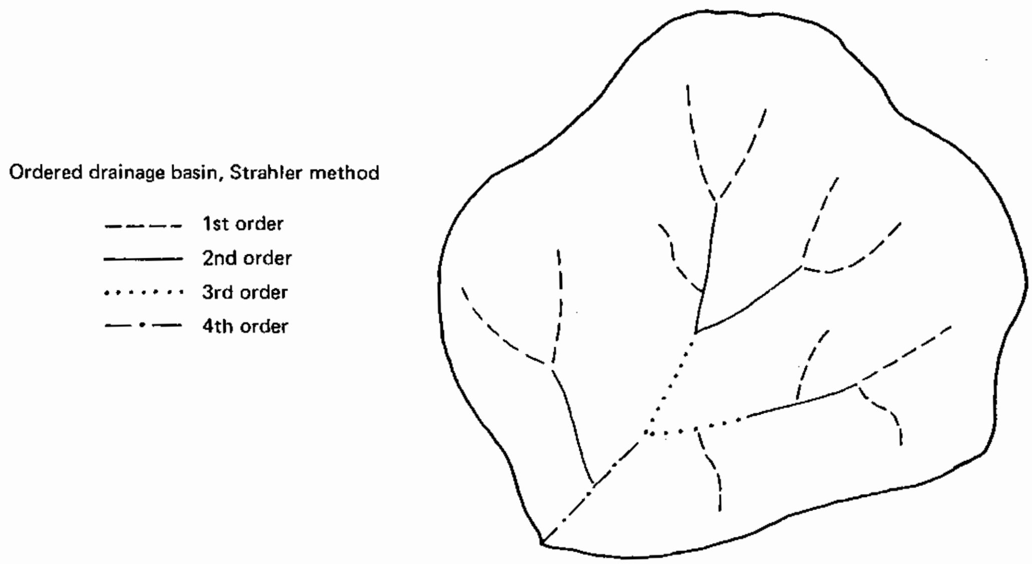


Figure 8.1. Sample basin showing stream orders according to the Strahler method.

By measuring various aspects of a basin, Horton and others determined a number of "laws" relating these aspects to order, as shown in Table 8.1. They found that morphometric characteristics have a regular relationship with order, such that when graphed on semi-logarithmic paper the points lie on a straight line.

Table 8.1. **Laws of Drainage Composition**

Law of stream numbers	$N_u = R_b^{s-u}$
Law of mean stream lengths	$\bar{L}_u = \bar{L}_1 R_L^{u-1}$
Law of basin areas	$\bar{A}_u = \bar{A}_1 R_a^{u-1}$
Law of total stream lengths	$\Sigma L_u = \bar{L}_1 R_b^{s-u} R_L^{u-1}$
Law of stream gradients	$\bar{S}_u = \bar{S}_1 R_s^{s-u}$

where N_u is number of streams of order u

R_b is bifurcation ratio

s is highest order of the basin

$\bar{L}_u$ is mean stream length of order u

R_L is stream length ratio

$\bar{A}_u$ is mean area of basin of order u

R_a is basin area ratio

ΣL_u is total stream lengths in a basin of order u

$\bar{S}_u$ is mean gradient of stream of order u

R_s is stream gradient ratio

Horton also devised a measure of the amount of dissection of the watershed which he called the drainage density, defined as the total of all stream lengths in the basin divided by the area of the basin, or

$$D = \frac{\Sigma L}{A}$$

OBJECTIVES

To carry out a partial drainage basin analysis (a complete Horton analysis of a large basin is tedious and unprofitable at your stage). To examine the validity of the first three of Horton's laws for a particular watershed.

PROCEDURE

The drainage net (Fig. 8.2) is part of the Buffalo River basin, Arkansas, taken from the Big Flat quadrangle. Precipitation over the basin averages approximately 46 inches per year, with 33% of this becoming runoff. The basin lies on the Ozark Plateau which is here carved on the Carboniferous cherty Boone limestone.

1. Examine the topographic map of the watershed and answer the questions asked.

2. For the drainage net given in Figure 8.2, determine the order of each stream segment according to Strahler's definition (Fig. 8.1). Count the number of streams of each order. Plot number of streams against order on arithmetic graph paper, with order plotted on the horizontal axis. Now plot the log of the number of streams against order. Draw a best fit line by eye.

3. Measure the length of each stream segment. Total the lengths by order and determine the mean length of each order. Plot the log of the mean stream length of each order against order. Draw a best fit line by eye. Plot log of the total lengths of each order against order.

4. Measure the total area of your watershed with a planimeter. Determine the drainage density as length of stream per unit area according to the formula given.

5. Determine the bifurcation ratio of the basin by calculating the least squares regression equation for the plot of log number of streams against order.

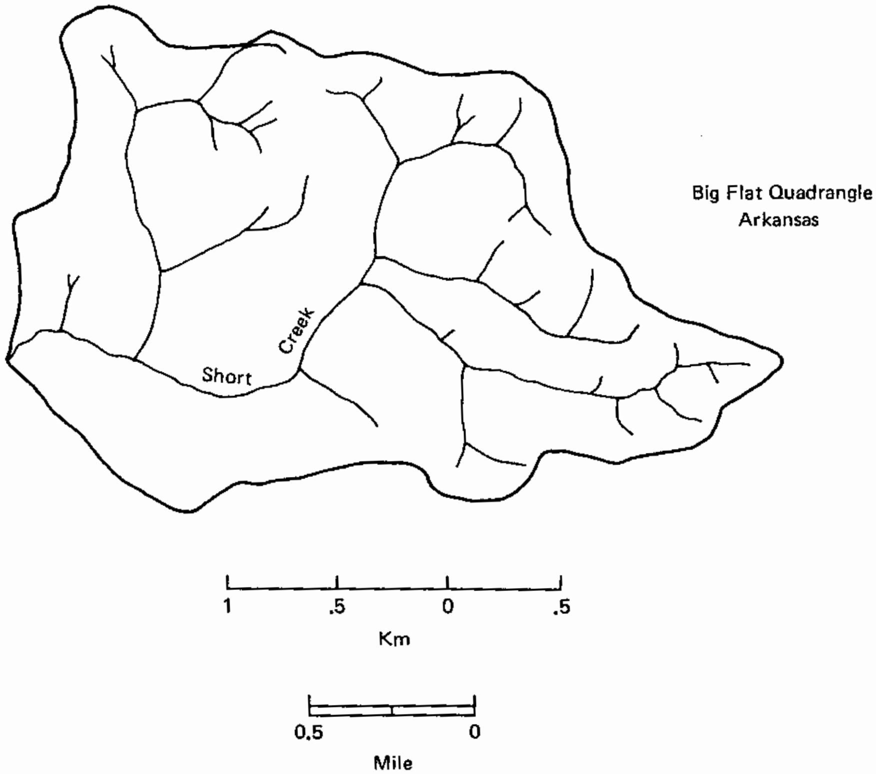


Figure 8.2. Drainage net, Short Creek, Big Flat quadrangle, Arkansas.

The Least Squares Regression Line

The simplest approximating curve for the points graphed is a straight line of the type

$$Y = a + bX$$

where a is the Y intercept and b is the slope of the line. If we have any two points (X_1, Y_1) and (X_2, Y_2) on the line we can determine the slope by substituting:

$$Y - Y_1 = \frac{Y_2 - Y_1}{X_2 - X_1}(X - X_1)$$

or

$$Y - Y_1 = b(X - X_1)$$

where

$$\frac{Y_2 - Y_1}{X_2 - X_1} = b \text{ (the slope of the line)}$$

We could draw the line by eye, but to avoid any bias in judgment we need an objective definition of the best fitting line. Figure 8.3 represents a series of data points $(X_1, Y_1), (X_2, Y_2), \ldots, (X_n, Y_n)$. For a given value of X, say X_1, there will be a difference between the real value of Y_1 and the value as determined from the curve. We denote this difference as D_1 (also referred to as the deviation, error or residual) which may be positive, negative, or zero. Thus for all values of X_i we can obtain deviations of the corresponding Y_i, denoting them as D_i.

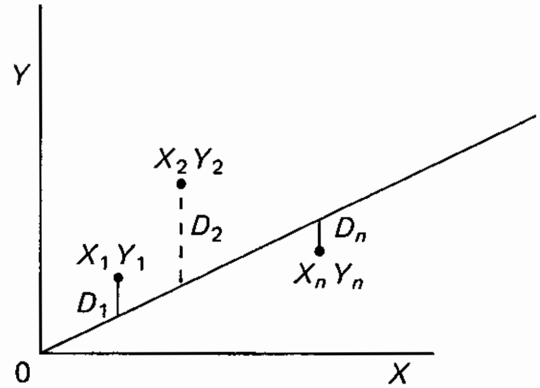


Figure 8.3. Least squares fit.

A measure of the fit of a curve to the given data is provided by deviations from the curve, $D_1^2 + D_2^2 + \ldots D_n^2$. If the total deviation is small the fit is good, if it is large the fit is not so good. Therefore, we can define the best fitting curve as that curve where the sum of the deviations from the line is a minimum.

$$D_1^2 + D_2^2 + \ldots + D_n^2 \text{ is a minimum}$$

A curve which fits the data in this sense is called the least square curve. The equation of the least square line is of the type

$$Y = a + bX$$

for a set of X and Y points. The constants a and b can be determined by solving the simultaneous equations

$$\Sigma Y = aN + b\Sigma X$$
$$\Sigma XY = a\Sigma X + b\Sigma X^2$$

which are called the normal equations for the least square line.

Least Squares Regression of Semi-logarithmic Data

In our problem we are looking for a best fit of a curve where the log number of streams is plotted against order. Thus in the least squares line

$$Y = a + bX$$

log of Y is substituted for Y. Our simple straight line regression equation is thus

$$\log Y = a + bX$$

or for procedure 5:

$$\log N_u = a - bu$$

($-b$ since the slope of the line in our problem is negative). So the corresponding simultaneous equations to be solved for the straight line are:

$$\Sigma \log Y = na + b\Sigma X$$
$$\Sigma(X \cdot \log Y) = a\Sigma X + b\Sigma X^2$$

8 SHEET 1

NAME ____

BIG FLAT, ARKANSAS (Map on page 101)

Scale: Contour Interval:

Drainage pattern:

What is the variation in relief?

What is the stage of dissection?

What geomorphic processes are at work?

What evidence is there on the map of the underlying rock type?

Sketch a profile from Leatherwood Creek south to the center of section 24 and indicate the rock layers on it.

What is the gradient of Short Creek?

What evidence is there that the main river (Buffalo River) has a widely fluctuating discharge?

 NAME ______________________

Watershed ______________________ Quadrangle ______________________

Order	Number of streams	Total length miles	Mean length miles
1			
2			
3			
4			

Total stream lengths ______________________

Basin area (sq. mi.) ______________________

Drainage density ______________________

CALCULATION OF BIFURCATION RATIO

Order X	Log no. streams log Y	$X \cdot \log Y$	X^2	
				total

Normal equations to be solved simultaneously:

$$\Sigma \log Y = na + b\Sigma X$$
$$\Sigma(X \log Y) = a\Sigma X + b\Sigma X^2$$

Substitute in these equations and solve for a and b.

8 SHEET 3 NAME ______________________________

CALCULATIONS:

Since by the law of stream numbers $N_u = R_b{}^{s-u}$

taking the logs of both sides $\log N_u = s \log R_b - u \log R_b$

let $a = s \log R_b$ and $b = \log R_b$

then $\log N_u = a - bu$

and $R_b = \log^{-1} b$, i.e., bifurcation ratio equals the antilog of the slope, b.

Thus, in your problem:

$\log N_u = a - bu$ is:

and since R_b is the antilog of b, the bifurcation ratio is:
(in determining the antilog ignore the minus sign which refers to the slope of the line, not the value of b.)

ANALYSIS AND CONCLUSIONS

Examine the results you have obtained. Do your graphs illustrate straight line relationships? Does your watershed conform to the laws of stream order and stream lengths? Explain any deviations from the law which are present. How do you think rock type might affect deviations? How would structure affect the relationships?

Discuss the effect of a drier climate on bifurcation ratio (i.e., on the number of tributaries in a stream system). What effect would a drier climate have on stream lengths? On drainage density?

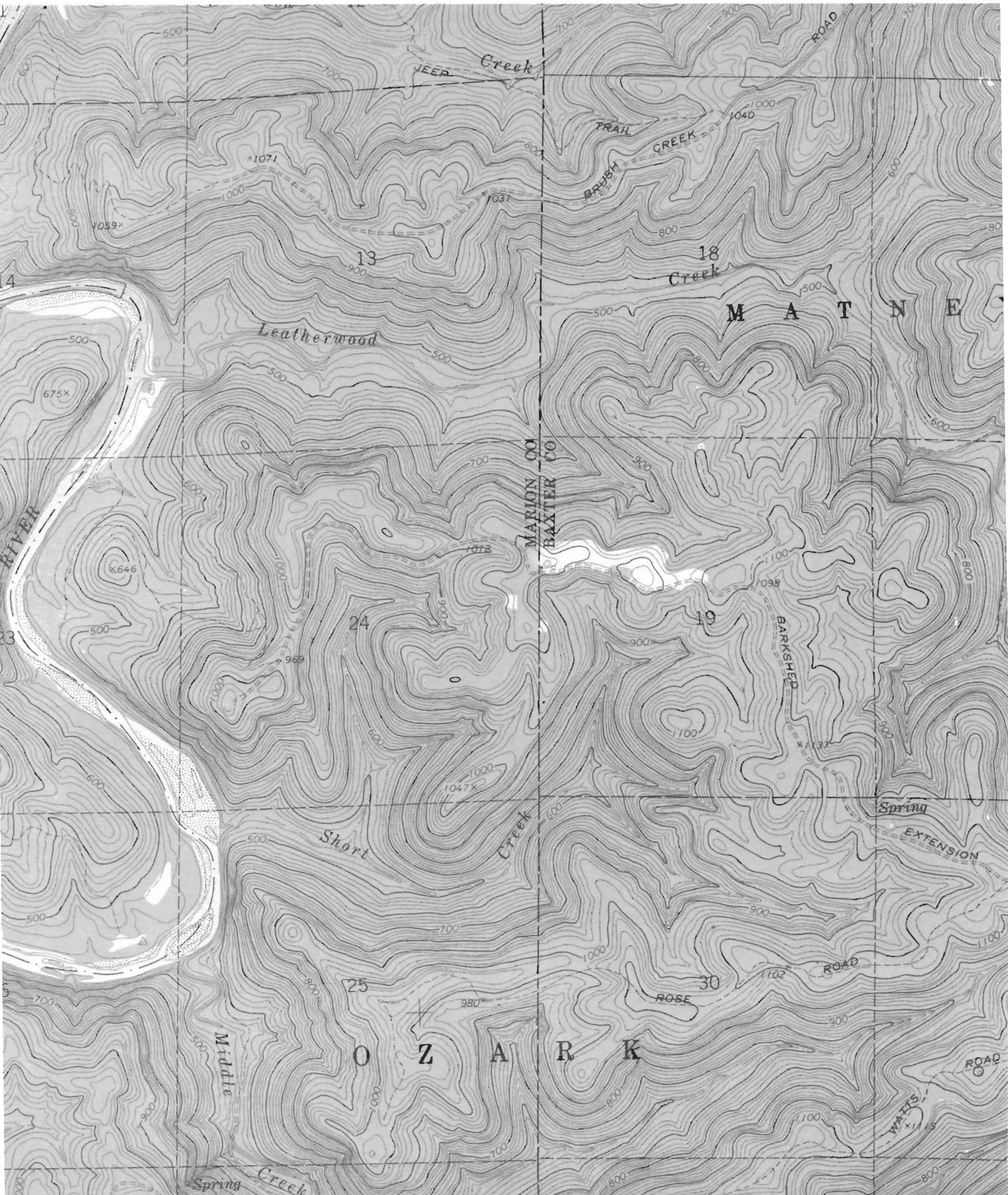
Would you expect a higher drainage density on sandstone or on shale? Longer stream lengths on sandstone or shale? Higher bifurcation ratios on sandstone or shale? Explain your answer in each case.

Could drainage density, bifurcation ratio or some other measure such as number of streams per unit area be used as a quantitative assessment of stage?

Organize your answers to these questions as a unified analysis with conclusions as to the value of a Horton drainage analysis.

REFERENCE

Morisawa, M. (1964) *Streams: Their Dynamics and Morphology.* McGraw-Hill Book Company.



BIG FLAT, ARK.
N3600—W9222.5/7.5
1966
SCALE 1:24 000
OZARK
NATIONAL FOREST
MATNE
MARION CO
BAXTER CO
RIVER
Leatherwood
Creek
Short
Creek
Middle
Spring
Creek
North
BRUSH CREEK
TRAIL
JEEP
Creek
ROAD
BARKSHED
Spring
EXTENSION
ROSE
ROAD
WATTS
ROAD
ROAD