Chapter 1

Bulk Properties of Rock

1.1 Bulk Density

Suppose that a sample of rock is under examination. Perhaps the sample was cut from rock in a quarry, or from an outcrop produced in the course of an engineering project. Call the volume of the sample V; i.e., V is the volume enclosed by the outer surface of the sample. The volume can be calculated from the dimensions of the sample if it has a shape for which there is a formula for the volume (e.g., a cylinder). If the sample has an irregular shape, then the volume may be harder to determine. It might be found from the displacement of water in a graduated cylinder into which the sample is immersed. Allowance may have to be made for any water absorbed by the sample.

On the other hand, the mass M of the sample is easy to measure, with a suitable balance, for example.

From the mass M and volume V the *density* ρ of the sample can be calculated. By definition

$$\rho = \frac{M}{V}. \tag{1.1}$$

The density defined by Eq.(1.1) is the average density of the sample: there is no way to tell from just the total mass and volume what is the internal distribution of density in the sample. This average density is also known as the *bulk density* .

In SI base units, mass is in kg (kilogram)and volume in m^3 (cubic meter), so that, by Eq. (1.1), the units of density are kg/m^3. In the cgs set of metric units, the unit of mass is g (gram) and of volume cm^3 (cubic centimeter); the density therefore has units g/cm^3. For future use, note that

$$1 \text{ kg/m}^3 = (1000 \text{ g})/(100 \text{ cm})^3,$$
$$1 \text{ kg/m}^3 = 1 \text{ X } 10^{-3} \text{ g/cm}^3,$$
$$1 \text{ g/cm}^3 = 1 \text{ X } 10^3 \text{ kg/m}^3.$$

Most of the rocks found near the surface of the Earth have densities in the range from 1.5 g/cm^3 to 3 g/cm^3.

Specific Gravity

The densities of substances making up rocks, or of a rock itself, are sometimes expressed in terms of the density ρ_w of liquid water. The *specific gravity* G of a substance is defined by

$$G = \frac{\rho}{\rho_w},$$ (1.2)

where ρ is the density of the substance. Since G is the ratio of two densities, the density units cancel, so that G has no units. (This requires that both densities in Eq.(1.2) must be expressed in the same units, either both kg/m^3 or g/cm^3; it would lead to meaningless confusion if one density is in kg/m^3 and the other g/cm^3.)

For the density of water it is common to use $\rho_w = 1.00$ g/cm^3. The density of water varies slightly with temperature (as does the density of other substances), but this variation can often (not always) be ignored. In the present work, the density of water will be taken as 1000 kg/m^3 exactly for the purpose of calculating specific gravity.

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EXAMPLE 1
A block of rock with edge lengths 85.5 cm, 79.0 cm, 43.8 cm has a mass of 953 kg. Find the specific gravity of the rock.

Some of the data are in SI base units (the mass), the rest are in cgs units (the edge lengths). Calculations should be done either with all base units or all cgs units. In engineering, the base units are more commonly used. (But not always: sometimes SI base units and cgs units are mixed!) Using all SI base units means that the edge lengths must be converted to meters. But this is easy since 1 m = 100 cm. By "block" of rock is implied a rectangular block whose volume is the product of the edge lengths. Therefore,

$$V = (0.855 \text{ m})(0.790 \text{ m})(0.438 \text{ m}),$$
$$V = 0.2958 \text{ m}^3.$$

By Eq.(1.1), the density is

$$\rho = \frac{M}{V},$$

$$\rho = \frac{953 \text{ kg}}{0.2958 \text{ m}^3},$$

$$\rho = 3222 \text{ kg/m}^3.$$

Now use Eq.(1.2) to find the specific gravity G. Since SI base units are being used here, the density of water ρ_w must be entered in these units. Therefore,

$$G = \frac{\rho}{\rho_w},$$

$$G = \frac{3222 \text{ kg/m}^3}{1000 \text{ kg/m}^3},$$

$$G = 3.22.$$

Since the data has 3 significant figures (sig fig), the final result can be given to no more than 3 sig fig, although 1 extra sig fig is carried within the calculation to guard against round-off error. As mentioned previously, the density of water is considered to be exact.

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1.3 Unit Weight

The weight W of an object is the gravitational force exerted on it by the rest of the planet Earth. Dividing the weight W of an object by its volume V yields the *unit weight* γ of the object or of the material of which the object is made; that is,

$$\gamma = \frac{W}{V}. \tag{1.3}$$

The SI base unit of weight is the newton (N). Since the base unit of volume is m^3, the SI base unit of unit weight is N/m^3. (Note the two uses of the word *unit* in the last sentence.) The cgs unit of weight, the dyne, will not be used in this book; therefore, cgs units of unit weight will not be encountered in the present work.

Looking back at Eq.(1.1), it can be seen that the unit weight is defined very like the bulk density, except that the weight W replaces the mass M. (For this reason, unit weight is also known as *weight density*.) There is a relation between weight and mass. From physics,

$$W = Mg, \tag{1.4}$$

where g is the acceleration due to gravity (often called simply *gravity*). Hence,

$$\gamma = Mg/V,$$

$$\gamma = (M/V)g,$$

$$\gamma = \rho g. \tag{1.5}$$

Equation (1.5) can be used to calculate unit weight from density, and vice versa. The numerical value of g to be used is

$$g = 9.8 \text{ m/s}^2.$$

This value is often adopted as the value of gravity averaged over the surface of the Earth. For the purpose of evaluating the significant figures in any calculation, this value for g shall be considered exact.

Note that Eq.(1.2) for specific gravity can be written in terms of unit weight:

$$G = \rho/\rho_w,$$

$$G = \rho g/\rho_w g,$$

$$G = \gamma/\gamma_w. \tag{1.6}$$

The unit weight of water is, by Eq.(1.5),

$$\gamma_w = \rho_w g,$$

$$\gamma_w = (1000 \text{ kg/m}^3)(9.8 \text{ m/s}^2),$$

$$\gamma_w = 9.8 \text{ kN/m}^3.$$

The SI prefix k stands for 1×10^3.

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EXAMPLE 2
Calculate the density in g/cm^3 of a rock with unit weight 27.6 kN/m^3.

As noted just above, the SI prefix k means a factor of 1000. By Eq.(1.5),

$$\gamma = \rho g,$$
$$27.6 \times 10^3 \text{ N/m}^3 = \rho(9.8 \text{ m/s}^2),$$
$$\rho = 2820 \text{ kg/m}^3,$$
$$\rho = 2.82 \text{ g/cm}^3.$$

The answer is given to 3 sig fig since the data is given to 3 sig fig (remember that the value $g = 9.8 \text{ m/s}^2$ is considered to be of infinite precision). Also, the density conversion factor between kg/m^3 and g/cm^3 is used in the last step.

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1.4 Porosity

Under a microscope, most types of rock are seen to contain small open spaces, called *pores*. These pores can originate in various ways. For example, many sedimentary rocks seem to be assembled from many small, solid particles, called *grains*. These grains are irregularly shaped. However, unlike a jigsaw puzzle in which the irregularly shaped pieces completely

interlock leaving no spaces, the grains in rock may not fit together perfectly, leaving the gaps called pores. See Fig.(1.1).
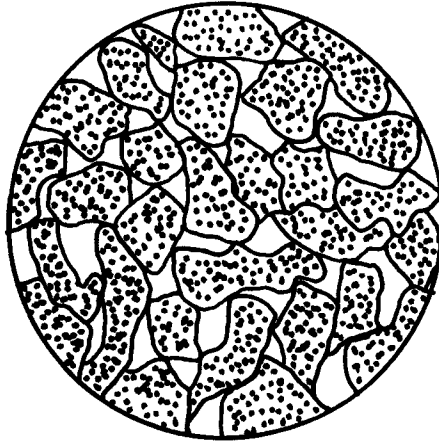
Fig.(1.1) Section of Rock Showing Grains Enclosing Pores.

Pores may simply be cracks in the rock, the result of mechanical or thermal forces exerted on the rock sometime in the past. They could be bubbles frozen into the rock when it solidified.

Whatever the origin of the pores, the truly solid part or parts of the rock are called the *grains* or the *matrix*. The terms grains and matrix often are used interchangeably.

To quantitatively express the degree of porosity (volume of pores vs. volume of rock), the *porosity* n is defined by

$$n = V_{\text{pores}}/V. \tag{1.7}$$

In Eq.(1.7), V_{pores} is the total volume of all the pores present in a rock sample whose volume is V. It is important to recognize that V is the volume of the rock sample as found in nature (in the field) and includes both the pores and the matrix; that is

$$V = V_{\text{pores}} + V_{\text{matrix}}, \tag{1.8}$$

where V_{matrix} is the total volume of all the truly solid portions of the rock. Put another way, as in the definition of bulk density, V is the volume that would be calculated from the external dimensions of the rock sample, with no regard for how much of the volume is occupied by grains or pores.

If the sample of rock being examined for porosity has been broken off from a much larger formation of the rock in the field, then the sample must be large enough to include a great many pores, to ensure that the sample is representative of the rock formation.

Since the total pore volume must be less than the volume of the rock (i.e., $V_{\text{pores}} < V$), it follows that $n < 1$. The situation $V_{\text{pores}} = V$ indicates just empty space ($V_{\text{matrix}} = 0$, by Eq.(1.8)), so that $n = 1$ is never realized for a real rock sample. On the other hand, in some very fine-grained rock, the grains can interlock like a jigsaw puzzle, yielding zero porosity. Hence, the values of porosity actually encountered are in the range $0 \leq n < 1$.

Often, porosity is expressed as a percent (%). However, in the equations in this book (such as Eq.(1.7)), it is always assumed that n is expressed as a decimal. Therefore, if n is given as a percent, it must be divided by 100 to obtain the corresponding decimal value before being used in any equation.

The density ρ of a rock defined by Eq.(1.1), and the unit weight γ (weight density) defined by Eq.(1.3), refer to the overall rock sample; i.e., the volume V in both of these equations encloses both grains and pores. These densities are average densities.

The density ρ_g of the grains (or matrix) is given by a relation analogous to Eq.(1.1), but applied to the grains:

$$\rho_g = \frac{M_{\text{grains}}}{V_{\text{grains}}}, \tag{1.9}$$

where M_{grains} is the total mass of the grains in the rock sample containing a volume V_{grains} of grains.

If the pores are unoccupied, or filled only with a gas, then $M_{\text{grains}} = M$, where M is the mass of the rock sample. (A gas contributes negligible weight to the sample.) By Eqs.(1.7) and (1.8),

$$V_{\text{grains}} = V - V_{\text{pores}},$$

$$V_{\text{grains}} = V - nV,$$

$$V_{\text{grains}} = (1 - n)V. \tag{1.10}$$

Substituting this into Eq.(1.9) and invoking Eq.(1.1) yields

$$\rho_g = \frac{M}{(1-n)V},$$

$$\rho_g = \frac{\rho}{1-n}. \tag{1.11}$$

This equation for the density ρ_g of the grains in terms of the bulk density ρ of the rock and of its porosity n applies only if the pores are either empty or occupied only by a gas. In practice, it is unlikely that the pores will be found to be truly empty (i.e., enclosing a vacuum). The term *empty* implies, therefore, that the pores are filled only with a gas (e.g., air, or methane).

An equation like Eq.(1.11) can be written in terms of unit weights, rather than mass densities. Multiply Eq.(1.11) by gravity g and use Eq.(1.5) to get

$$\gamma_g = \frac{\gamma}{1-n}. \tag{1.12}$$

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EXAMPLE 3

A 0.885-m^3 block of sandstone has a mass of 1752 kg. When the block is crushed just sufficiently to close all the pores, which are empty, the volume of the rock becomes 0.584 m^3. Find (a) the porosity of the sandstone and (b) the density of the grains. (Assume that the density of the grains is not changed in the crushing process.)

(a) The original block has a volume $V = 0.885$ m^3. The volume of the crushed rock must equal the volume of all the grains in the original block, since the crushed rock has zero pore volume. That is, in the original block, $V_{grains} = 0.584$ m^3. Therefore, the volume of the pores in the original block is, by Eq.(1.8),

$$V = V_{pores} + V_{grains},$$
$$0.885 \text{ m}^3 = V_{pores} + 0.584 \text{ m}^3,$$
$$V_{pores} = 0.301 \text{ m}^3.$$

Now calculate the porosity by Eq.(1.7):

$$n = V_{pores}/V,$$
$$n = (0.301 \text{ m}^3)/(0.885 \text{ m}^3),$$
$$n = 0.340 \ (34.0\%).$$

(b) Since the pores are empty, the mass of the crushed rock is the same as that of the original block, 1752 kg. The volume of the crushed rock is 0.584 m^3. But the crushed rock is entirely grains, and therefore, by Eq.(1.9),

$$\rho_g = M_{grains}/V_{grains},$$
$$\rho_g = (1752 \text{ kg})/(0.584 \text{ m}^3),$$
$$\rho_g = 3000 \text{ kg/m}^3,$$
$$\rho_g = 3.00 \text{ g/cm}^3.$$

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1.5   Dry and Saturated Unit Weights

The pores of *in situ* rock (rock as found in the Earth, undisturbed by human activity) may be filled with gas or liquid. The densities of gases found in rocks are very much less than the densities of the grains or matrix of the rocks. This means that, as already mentioned, it is safe to ignore the contribution of the gas trapped in the pores to the total weight of a rock sample.

A similar statement cannot be made for liquids. The densities of the liquids commonly found in the pores of rocks, although less than the densities of the grains, are not very much

les. If the porosity of the rock is large enough, and a significant fraction of the pores contain liquid, then the weight (or mass) of the liquid is likely to be an appreciable part of the total weight (or mass) of the rock.

If all the pores in a rock sample are completely filled with liquid, then the rock is said to be *saturated*. If all the pores are empty, then the rock is said to be *dry*. Saturated rock can be rendered dry by heating the rock in an oven; at sufficiently high temperature the liquid vaporizes and the vapor is driven out of the rock.

An important relation is that between the unit weight γ_{sat} of a saturated rock sample, the unit weight γ_{dry} of the same sample when dry, and the unit weight γ_L of the liquid occupying the pores of the saturated sample.

(It may be tempting to write $\gamma_{sat} = \gamma_{dry} + \gamma_L$, but this is not correct because of the different volumes involved.)

To obtain the actual relation, note that the weight W_{sat} of the saturated rock sample is just the sum of the dry weight W_{dry} and the weight W_L of the liquid in the saturated rock:

$$W_{sat} = W_{dry} + W_L.$$

The volume V of the rock sample is the same whether it is dry or saturated (just as the volume of your car's gas tank, metal with one large pore, is the same whether the tank is empty or full). Dividing the preceding equation by V gives

$$\frac{W_{sat}}{V} = \frac{W_{dry}}{V} + \frac{W_L}{V},$$

$$\gamma_{sat} = \gamma_{dry} + \frac{W_L}{V}, \tag{1.13}$$

by Eq.(1.3). The unit weight of the liquid γ_L is

$$\gamma_L = \frac{W_L}{V_L}, \tag{1.14}$$

where V_L is the volume of the liquid with weight W_L. But, since the liquid fills all the pores,

$$V_L = V_{pores}. \tag{1.15}$$

Hence, by the definition of porosity n

$$V_L = nV. \tag{1.16}$$

Solve this last equation for V (easy!) and substitute into Eq.(1.13). Then use the definition of γ_L given in Eq.(1.14) to obtain

$$\gamma_{sat} = \gamma_{dry} + n\frac{W_L}{V_L},$$

$$\gamma_{sat} = \gamma_{dry} + n\gamma_L. \tag{1.17}$$

A similar relation holds between the mass densities. By Eq.(1.5), Eq.(1.17) becomes

$$\rho_{sat}g = \rho_{dry}g + n(\rho_L g),$$

$$\rho_{sat} = \rho_{dry} + n\rho_L. \tag{1.18}$$

Example 4 below describes how these relations can be used to determine the porosity of a rock sample by injecting it with mercury Hg. Liquid mercury is much denser than water: $G_{Hg} = 13.6$ compared with $G_w = 1$ for water. This implies that, even if the porosity of the rock sample is quite small, saturating the rock with mercury could change the unit weight significantly, making accurate laboratory measurements of the weights and their differences relatively easy. A disadvantage is that liquid mercury is a hazardous substance, mainly because of its vapor; great care must be exercised with its use.

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EXAMPLE 4

A test cylinder of rock has a diameter of 12.6 cm and a length of 14.0 cm. When dry its weight is 50.3 N. When saturated with mercury the weight of the sample is 62.8 N. The specific gravity of mercury is 13.6. Find the porosity of the rock.

The volume of the rock sample is

$$V = \pi D^2 L/4,$$

$$V = \pi(0.126 \text{ m})^2(0.140 \text{ m})/4,$$

$$V = 1.746 \text{ X } 10^{-3} \text{ m}^3.$$

Therefore, the dry and saturated unit weights are

$$\gamma_{dry} = W_{dry}/V,$$

$$\gamma_{dry} = (50.3 \text{ N})/(1.746 \text{ X } 10^{-3} \text{ m}^3),$$

$$\gamma_{dry} = 28.81 \text{ kN/m}^3;$$

$$\gamma_{sat} = W_{sat}/V,$$

$$\gamma_{sat} = (62.8 \text{ N})/(1.746 \text{ X } 10^{-3} \text{ m}^3),$$

$$\gamma_{sat} = 35.97 \text{ kN/m}^3.$$

The unit weight of the liquid mercury γ_L follows from Eq.(1.6):

$$\gamma_L = G_L\gamma_w,$$

$$\gamma_L = (13.6)(9.8 \text{ kN/m}^3),$$

$$\gamma_L = 133.3 \text{ kN/m}^3.$$

Note that the unit weight of the mercury is greater than the unit weight of the rock, whether dry or saturated. Now use Eq.(1.17) to solve for the porosity n. Note that the units of unit weight cancel, so that

$$\gamma_{sat} = \gamma_{dry} + n\gamma_L,$$

$$35.97 \text{ kN/m}^3 = 28.81 \text{ kN/m}^3 + n(133.3 \text{ kN/m}^3),$$

$$n = 0.0537 \ (5.37\%).$$

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1.6 Subsidence

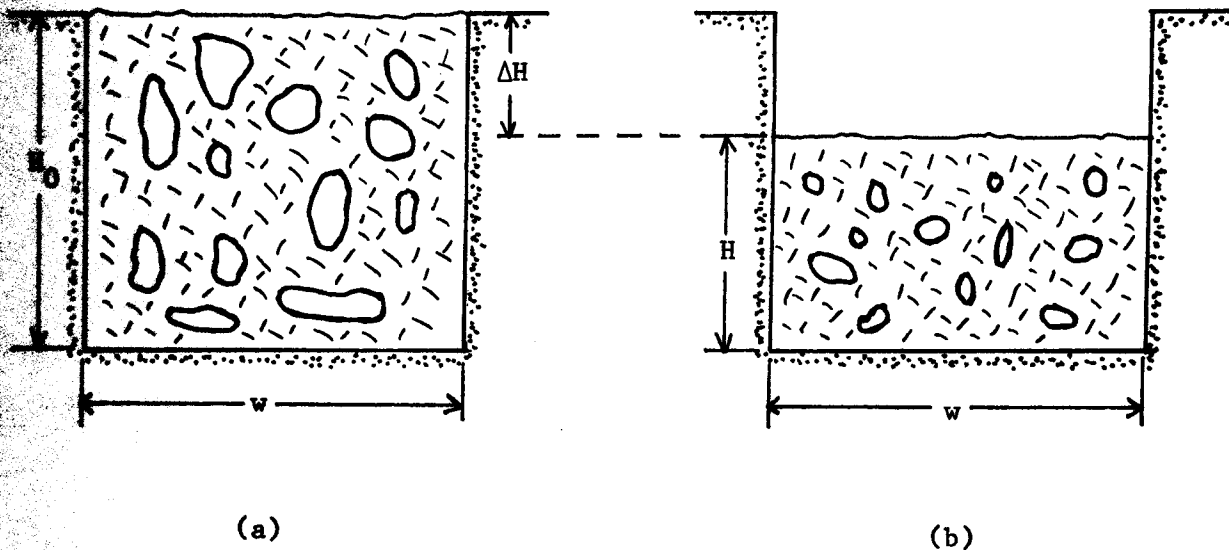(a)   (b)

Fig.(1.2) Consolidation and Subsidence

Suppose that loose, unconsolidated, rock (or soil) is dumped into a rectangular trench of length L, width w, and depth H_0, as shown in Fig.(1.2a). Initially the material, of porosity n_0 just fills the trench. However, because of an applied external load (such as the weight of a building constructed thereon), the material is compacted, or consolidated. That is, its pores partially collapse, squeezing out any water that might have occupied them (this requires that the water has a place to go, and that the consolidation is not so rapid that the water cannot get there). Eventually, the consolidation ceases, with the porosity reduced to $n < n_0$. It shall be presumed that the density of the matrix is unaffected; this is reasonable, since the force needed to shrink the pores is much less than that needed to densify the matrix. The material now occupies a region of smaller volume, so that it fills the trench only to a height $H < H_0$. This means that the surface of the material (the ground) is lowered by an amount $\Delta H = H - H_0$. See Fig.(1.2b). A lowering of the ground surface is called *subsidence* . It can occur for reasons other than a change in porosity, and can take place slowly or quickly.

The task now is to calculate the subsidence ΔH due to a reduction in porosity. To do this, write expressions for the volume of the matrix before and after the consolidation. Since, by assumption, the density of the matrix is unchanged, its volume is also (provided there is no loss of matrix). Hence, the two expressions must be equivalent. By the definition of

porosity, they are

$$V_{\text{matrix}} = (1 - n_0)(H_0 w L),$$

$$V_{\text{matrix}} = (1 - n)(H w L).$$

Setting these expressions equal gives

$$(1 - n_0)(H_0 w L) = (1 - n)(H w L),$$

$$(1 - n_0)H_0 = (1 - n)H,$$

$$(1 - n_0)H_0 = (1 - n)(H_0 - \Delta H),$$

$$\Delta H = H_0 [\frac{n_0 - n}{1 - n}]. \qquad (1.19)$$

Equation(1.19) applies only to a trench with a rectangular cross section, since the volume of the material was presumed to be given by the product of the three edge lengths.

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EXAMPLE 5

A layer of clay with a porosity of 47.0% and saturated with water is deposited into a rectangular trench 260 m long and 17.5 m wide to a depth of 2.72 m. Later, it is found that the clay has settled by 15.0 cm. Find the volume of water squeezed out of the clay.

The subsidence and the original height must be expressed in the same units, so that these units will cancel. Choosing meters, and remembering to express the porosity in decimal form, Eq.(1.19) becomes, after substitution of the data,

$$0.150 = 2.72[\frac{0.470 - n}{1 - n}],$$

$$n = 0.4391.$$

Since the clay was saturated, the volume V_w of water squeezed out equals the loss of pore space in the clay due to the compaction. Therefore,

$$V_w = n_0(H_0 w L) - n(H w L),$$

$$V_w = (n_0 H_0 - n H)w L.$$

Now $H = 2.72$ m - 0.15 m = 2.57 m, so that

$$V_w = [(0.470)(2.72 \text{ m}) - (0.4391)(2.57 \text{ m})](17.5 \text{ m})(260 \text{ m}),$$
$$V_w = 682 \text{ m}^3.$$

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1.7   Multimineral Rock

Suppose that the porosity of a particular rock specimen is to be found by measuring the bulk density ρ, the grain density ρ_g and then applying Eq.(1.11). The bulk density is easy to measure. What about the grain density?

If all of the grains in the rock are of the same mineral, and the density of the mineral as it occurs in nature is known (many have been measured in the laboratory), then the grain density simply equals the density of that mineral.

For a rock that contains several minerals, the value of ρ_g to use in equations like Eq.(1.11) is the average of the densities of the individual minerals present. As an example, consider a rock made up of three minerals, the densities of the minerals grains being denoted by ρ_1, ρ_2 and ρ_3. The bulk grain density ρ_g will not, in general, be simply $\frac{1}{3}(\rho_1 + \rho_2 + \rho_3)$, because the minerals may be present in different amounts. A *weighted* average must be used, the precise nature of which must now be deduced.

Let the total mass of all the grains in the rock sample be M_g and the total volume of all the grains V_g. By Eq.(1.9),

$$M_g = \rho_g V_g.$$

If M_1 be the total mass and V_1 the total volume of mineral 1 in the rock, with similar notation for the other two minerals present (assuming that the rock contains three minerals), then since

$$M_g = M_1 + M_2 + M_3,$$

it follows that

$$\rho_g V_g = \rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3,$$

$$\rho_g = \rho_1 \left(\frac{V_1}{V_g}\right) + \rho_2 \left(\frac{V_2}{V_g}\right) + \rho_3 \left(\frac{V_3}{V_g}\right).$$

Finally, write

$$f_1 = \frac{V_1}{V_g}, \tag{1.20}$$

for the fractional volume abundance of the first mineral, with similar notation for the other minerals), so that the bulk grain density becomes

$$\rho_g = f_1 \rho_1 + f_2 \rho_2 + f_3 \rho_3. \tag{1.21}$$

Equation (1.21) is the relation sought for the bulk grain density; it is a volume-weighted average of the densities of the individual minerals present.

Writing equations like Eq.(1.20) for the three minerals and then adding gives

$$f_1 + f_2 + f_3 = 1. \tag{1.22}$$

The volume abundances can be measured in the laboratory by examining a representative piece of the rock with a microscope powerful enough so that the individual grains can be

seen and distinguished from each other. The number of grains of each mineral present must then be counted and their total volume estimated.

Identifying a mineral by name does not necessarily specify its chemical composition. For example, a grain of the mineral olivine can contain some or all of the molecules Fe_2SiO_4, $FeMgSiO_4$, and Mg_2SiO_4. Different olivine grains will include these molecules with varying abundancies.

This leads to the expectation that different olivine grains have different densities. This is indeed the case. However, the variation in densities is fairly small, between about 3.2 g/cm^3 and 3.6 g/cm^3.

This is the situation for many other (but not all) minerals. Although the densities of their grains shows some variation, the variation is small enough that, to a good approximation, the average of these densities can be taken as the density of all grains of the mineral. Table (1.1) lists some minerals which show small variation in density and their average density.

Mineral	Density (g/cm^3)
Gypsum	2.35
Orthoclase	2.55
Chalcedony	2.62
Quartz	2.65
Plagioclase	2.70
Chlorite	2.80
Muscovite	2.85
Anhydrite	2.95
Pyroxene	3.40
Barite	4.45
Pyrite	5.05
Galena	7.54

Table (1.1) Average Density of Some Minerals

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EXAMPLE 6
A shale consists of 34.1% chlorite and 65.9% pyrite, and has a porosity of 38.8%. Find the bulk density of the shale.

First, use Eq.(1.21), suitably modified for a rock that consists of only two minerals, and Table (1.1) to calculate the bulk grain density:

$$\rho_g = f_{ch}\rho_{ch} + f_{py}\rho_{py},$$
$$\rho_g = (0.341)(2.80 \text{ g/cm}^3) + (0.659)(5.05 \text{ g/cm}^3),$$
$$\rho_g = 4.283 \text{ g/cm}^3.$$

From Eq.(1.11) the rock's bulk density ρ is

$$\rho = \rho_g(1 - n),$$
$$\rho = (4.283 \text{ g/cm}^3)(1 - 0.388),$$
$$\rho = 2.62 \text{ g/cm}^3.$$
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1.8   Triangular Composition Diagrams

The mineral composition of a rock is specified by giving the abundances of all the minerals that it contains. In many rocks, it is sufficient to give the abundances of just three minerals, either because the rock contains only three minerals, or that if it contains more, the abundances of the others are so small that they can be ignored.

For example, many igneous rocks contain mainly the minerals quartz, alkali feldspar, and plagioclase. The rocks are classified into different named groups in part on the range of abundances of these three minerals.

For these three-mineral rocks, it is fashionable to display the abundances of the minerals in the associated rock groups on a *triangular composition diagram*. In this way, the compositional relationships between the various associated rock groups can be visualized without examining tables of their mineral abundances.

To see how the compositional display can be constructed, it is only necessary to become convinced of this, otherwise rather obscure, property of equilateral triangles: From any point inside the triangle, draw perpendiculars to the three median lines; the sum of the three distances from the midpoint of each side to the foot of the perpendicular drawn to it equals the length of a median line.

Figure (1.3) shows an equilateral triangle with one vertex at the origin of an x,y coordinate system. Each side of the triangle has the same length L and each vertex angle is $60°$. The x,y coordinates of the vertices and of the midpoints of the three sides are given. An arbitrarily selected interior point is shown, labelled with its coordinates x_p, y_p. The dashed lines r_1, r_2, r_3 mark the perpendiculars from the interior point to the three median lines, and also represent the lengths of the perpendiculars. The distances from the foot of each perpendicular to the midpoint of the side of the triangle that the associated median line is drawn to are D_1, D_2, D_3.

Now the length of each median line is $L \cos 30° = L\sqrt{3}/2$. Hence, the property referred to above, in equation form, is

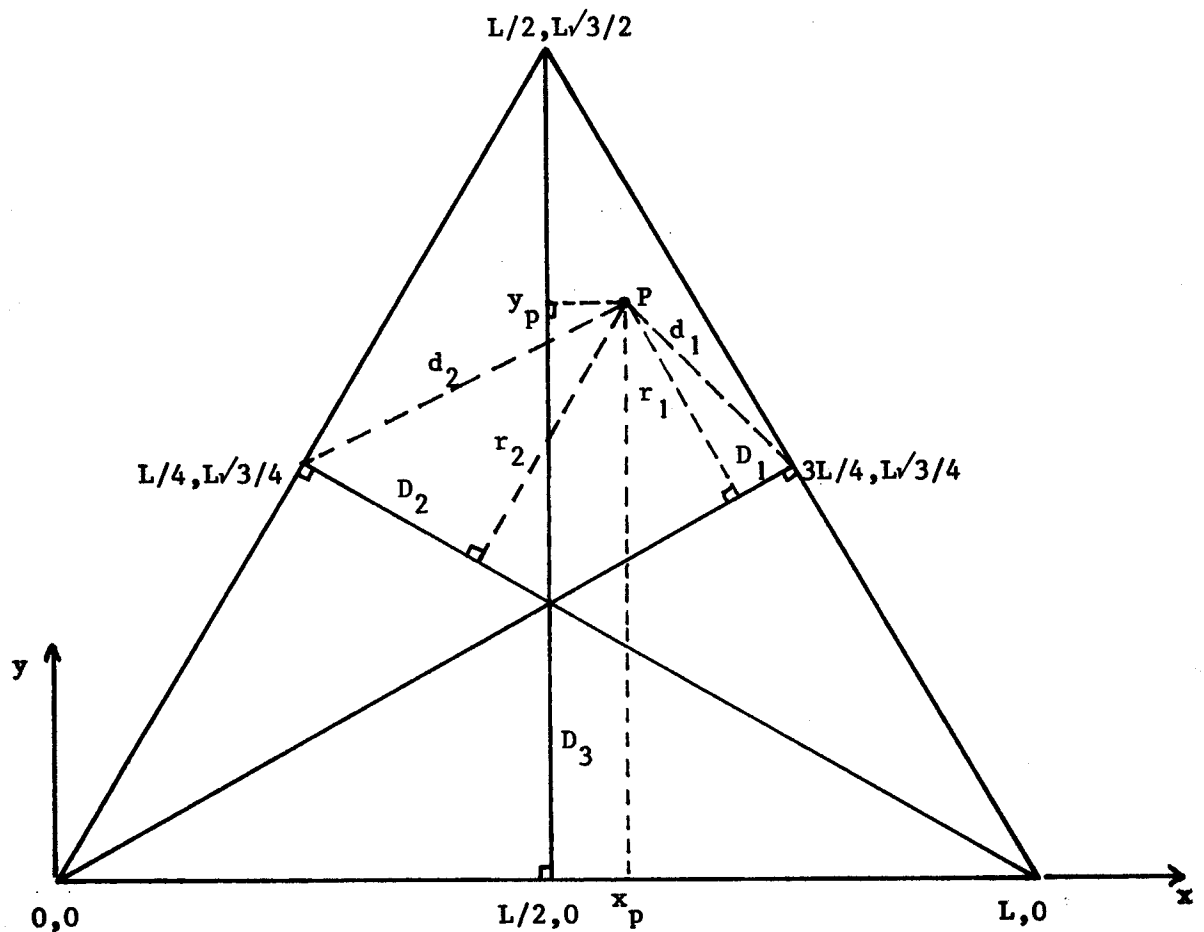$$D_1 + D_2 + D_3 = L\sqrt{3}/2. \tag{1.23}$$

Fig.(1.3) An Equilateral Triangle

To establish this, first recall from algebra and analytic geometry that the perpendicular distance r from a point on the x, y plane with coordinates x_p, y_p to the line whose equation is $y = mx + b$ is given by

$$r = \pm \frac{y_p - (mx_p + b)}{\sqrt{m^2 + 1}}, \qquad (1.24)$$

where the choice indicated by $\pm$ is made such that r is positive.

Determine D_1 first. To do this, draw the line marked, and of length, d_1 from the interior point to the midpoint of the side intersected by the median line to which r_1 is drawn; see

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EXAMPLE 7

Draw a triangular composition diagram for rocks consisting of the three minerals called R, S, and T. On the diagram, locate rocks with compositions (a) 50% R, 50% S; (b) 70%R, 10% S; (c) 60% T.

Draw an equilateral triangle; label the vertices R, S, T in any order. Draw the three median lines and divide into convenient intervals from 0% at the intersection with the side to 100% at the vertex. See Fig.(1.5). (a) A composition 50% R, 50% S means that the abundance of T is 100% - 50% - 50% = 0. Hence, the composition point falls at the 0% mark opposite the T vertex, on the side of the triangle that connects the R and S vertices. (b) Through the medians from the R and S vertices, draw perpendiculars through the marks at the proper percentages. The composition point sought must be at the intersection of these perpendiculars. That point will automatically indicate a T abundance of 100% - 70% - 10% = 20%. (c) All rocks with composition 60% T must lie on the perpendicular through the T median at the 60% mark. Since neither the R nor S composition is given, a specific point on this perpendicular cannot be identified.
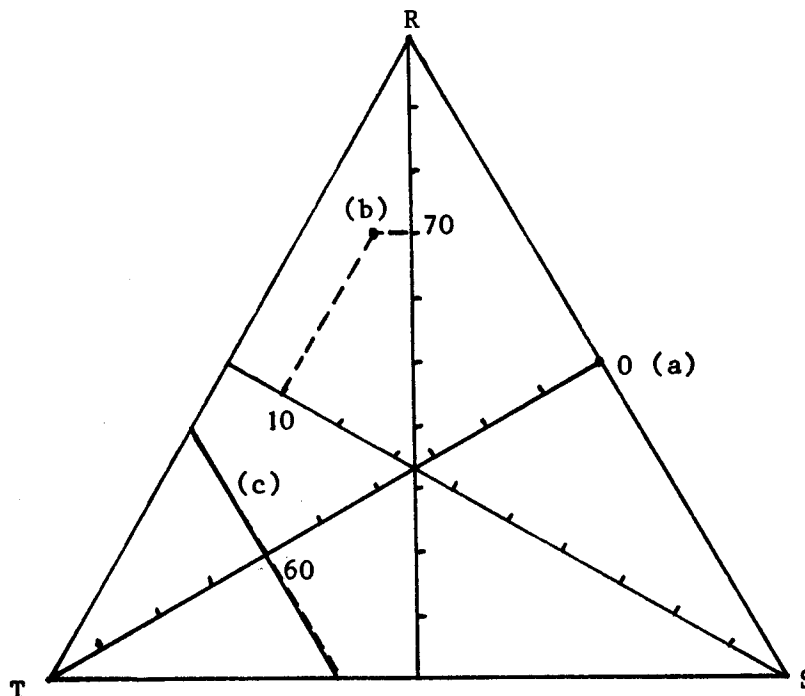

Fig.(1.5) Example 7

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