ALGEBRAIC PROPERTIES AND FORMULAS

Properties of Inequalities

If $a < b$, then $a + c < b + c$
If $a < b$ and $b < c$, then $a < c$
If $a < b$ and $c > 0$, then $ac < bc$
If $a < b$ and $c < 0$, then $ac > bc$
If $ab > 0$ and $a < b$, then $\frac{1}{a} > \frac{1}{b}$

Properties of Absolute Value

$|a| = a$ if $a \geq 0$
$|a| = -a$ if $a < 0$
$|-a| = |a|$
$|ab| = |a||b|$
$|a + b| \leq |a| + |b|$
$|a|^2 = a^2$

Properties of Exponents

$a \neq 0$ and m and n are integers

$$a^n = \underbrace{a \cdot a \cdot a \cdot \ldots a}_{n \text{ factors}} \text{ if } n > 0$$

$a^{1/n} =$ the n^{th} root of a

$$a^{-n} = \frac{1}{a^n}$$

$$a^{m/n} = (a^{1/n})^m$$

If p and q are positive rational numbers

$$(a^p)^q = a^{p \cdot q} = (a^{1/p})^q$$

$$a^{p/q} = (a^{1/q})^p$$

$$a^p a^q = a^{p+q}$$

$$\frac{a^p}{a^q} = a^{p-q}$$

$$(ab)^p = a^p b^p$$

$$\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{1}{(a/b)} = \frac{b}{a}$$

Properties of Polynomials

$$(x + y)^2 = x^2 + 2xy + y^2$$

$$(x - y)^2 = x^2 - 2xy + y^2$$

$$(x + y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

$$(x - y)^3 = x^3 - 3x^2y + 3xy^2 - y^3$$

$$x^2 - y^2 = (x + y)(x - y)$$

$$x^3 + y^3 = (x + y)(x^2 - xy + y^2)$$

$$x^3 - y^3 = (x - y)(x^2 + xy + y^2)$$

Properties of Logarithms

Suppose $a \neq 1$, $a > 0$, $x > 0$, and $w > 0$.

$$a^{\log_a x} = x$$

$$\log_a a^x = x$$

$$\log_a a = 1$$

$$\log_a 1 = 0$$

$$\log_a xw = \log_a x + \log_a w$$

$$\log_a x^r = r \log_a x$$

$$\log_a \frac{x}{w} = \log_a x - \log_a w$$

$$\log x = \log_{10} x$$

$$\ln x = \log_e x$$

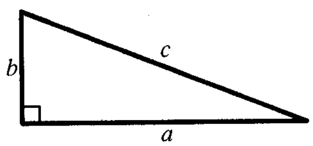
The Quadratic Formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are the solutions to
$ax^2 + bx + c = 0$

The Binomial Formula

$$(x + y)^n = x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + \cdots + \binom{n}{j}x^{n-j}y^j + \cdots + \binom{n}{n-1}xy^{n-1} + y^n$$

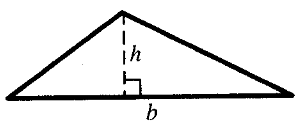
GEOMETRIC FORMULAS



Right Triangle

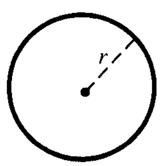
- **Triangles**

  Pythagorean Theorem $a^2 + b^2 = c^2$



Any Triangle

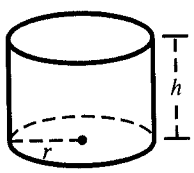
  Area $A = \frac{1}{2}bh$



- **Circles**

  Area $A = \pi r^2$

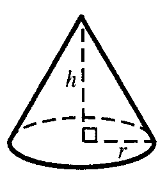
  Circumference $C = 2\pi r$



- **Cylinders**

  Surface Area $S = 2\pi r^2 + 2\pi rh$

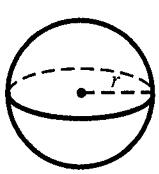
  Volume $V = \pi r^2 h$



- **Cones**

  Surface Area $S = \pi r^2 + \pi r\sqrt{r^2 + h^2}$

  Volume $V = \frac{1}{3}\pi r^2 h$

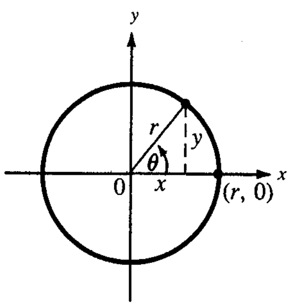


- **Spheres**

  Surface Area $S = 4\pi r^2$

  Volume $V = \frac{4}{3}\pi r^3$

TRIGONOMETRIC FUNCTIONS AND LAWS



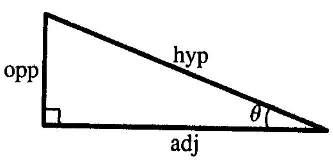
- **Definitions Based on the Circle**

$\cos\theta = \dfrac{x}{r}$ $\sec\theta = \dfrac{r}{x}$

$\sin\theta = \dfrac{y}{r}$ $\csc\theta = \dfrac{r}{y}$

$\tan\theta = \dfrac{y}{x}$ $\cot\theta = \dfrac{x}{y}$

For the unit circle, $r = 1$

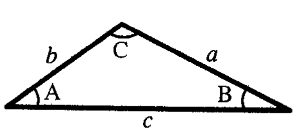


- **Definitions Based on the Right Triangle**

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ $\sec\theta = \dfrac{\text{hyp}}{\text{adj}}$

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ $\csc\theta = \dfrac{\text{hyp}}{\text{opp}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$ $\cot\theta = \dfrac{\text{adj}}{\text{opp}}$



- **Law of Sines**

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

- **Law of Cosines**

$$a^2 = b^2 + c^2 - 2\,bc\cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2\,bc}$$

TRIGONOMETRIC IDENTITIES

Identities that follow from the Definitions

$$\tan x = \frac{\sin x}{\cos x} \qquad \cot x = \frac{\cos x}{\sin x} = \frac{1}{\tan x}$$

$$\sec x = \frac{1}{\cos x} \qquad \csc x = \frac{1}{\sin x}$$

Circular or Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Even-Odd Identities

$\cos(-x) = \cos x \qquad \sin(-x) = -\sin x$

$\tan(-x) = -\tan x \qquad \cot(-x) = -\cot x$

$\sec(-x) = \sec x \qquad \csc(-x) = -\csc x$

Sum and Difference Identities

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Double-Angle Identities

$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1 = 1 - 2\sin^2 x$$

$$\sin 2x = 2\sin x\cos x \qquad \tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$

Half-Angle Identities

$$\sin\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos\frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{1 + \cos x}}$$
$$= \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

Product-to-Sum Identities

$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$

$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$

$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha + \beta) - \sin(\alpha - \beta)]$

$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha + \beta) + \cos(\alpha - \beta)]$

Sum-to-Product Identities

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha + \beta}{2}\sin\frac{\alpha - \beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha + \beta}{2}\cos\frac{\alpha - \beta}{2}$$

$$\cos\alpha - \cos\beta = 2\sin\frac{\alpha + \beta}{2}\sin\frac{\beta - \alpha}{2}$$