I. Introduction and Basic Concepts

A. Stress: force applied to rock unit, that results in deformation (strain)

B. Definitions

1. Force: vector with magnitude and direction

a. compressional vs. tensional forces in geology
(1) squeezing vs. pulling apart

b. magnitude = how much force?
c. direction = direction of force?

d. Force = Mass x Acceleration

2. Traction = force distributed per unit area

a. given a constant magnitude...

(1) > area, < traction (lesser concentration of force)
(2) < area, > traction (greater concentration of force

b. Stress = "traction" = Force / Area

(1) e.g. force applied to a fracture plane or bedding plane

3. Force Components

a. 2-D Analysis

(1) Force may be broken into 2 vector components oriented at right angles

b. 3-D Analysis

(1) Force may be broken into 3 vector components oriented at right angles

c. Force distributed over an area
(1) Force component normal to surface ("normal stress)
(2) Force component parallel to surface ("shear stress")

4. Surface Stress Equilibrium

a. Traction force applied to surface

(1) Equilibrium condition: a pair of equal and opposite tractions acting across a surface of given orientation

5. Vector Review

a. Vector: quantity with magnitude and direction

(1) e.g. Velocity, Force

(a) e.g. car travels 40 mi/hr in east direction

(2) Graphical depiction

(a) arrow shows direction
(b) length of arrow scaled to magnitude

b. "Scalar Quantity": magnitude only

(a) e.g. area, temperature, density

c. Vectors in 2-D

(1) Parallelogram method of vector resolution

(a) Vector addition: V + W = R

d. Vectors in 3-D

(1) Orthogonal Cartesian Coordinate System

(a) x-y-z axes mutally perpendicular (also known as x_1, x_2 and x_3 respectively)

(2) Resolution of force F in 3-D

(a) F = F1+F2+F3 where F1, F2 and F3 = force components parallel to x,y,z reference axes respectively

6. Remember Your Trigonometry!!!
a. Triangles

(1) All interior angles of any triangle must = 180 degrees
(2) Right Triangle: one of the angles of triangle = 90 degrees

b. Right Triangles and Trig. Functions

(1) theta = θ = given interior angle of right triangle, not the 90 degree angle
(2) "hypotenuse" = line opposite right angle of right triangle
(3) "adjacent" = line forming ray of angle θ
(4) "opposite" = line opposite angle θ
(5) 2 θ = 2 times the angle of θ

c. Basic Trig. Functions

(1) Sin θ = length opposite / length hypotenuse

(2) Cos θ = length adjacent / length hypotenuse
(3) Tan θ = Sinθ/Cosθ = length opposite / length adj.
(4) CSC θ = 1/Sinθ = hyp/opp.
(5) Sec θ = 1/Cosθ = hyp./adj.
(6) Cot θ = 1/Tan = adj./opp. = cosθ/sinθ

7. Units of Force and Stress

a. Force Units (F = Mass x Acceleration)

(1) Newton = amount of force required to accelerate 1 kilogram of mass at 1 meter per second per second
(2) 1 N = 1 kg m/sec^2 = 0.225 lb (in english system)

b. Stress Units (Stress = Force / Area)
(1) Units = N/m^2

(a) 1 N/m^2 = 1 Pascal (Pa)
(b) 1 MPa = 1 megapascal = 10^6 Pa = 10^6 N/m^2
(c) 1MPa = 10 bars = 0.01 kb

(2) E.g. atmospheric pressure at sea level = ~ 1000 mb = 1 bar = 0.1 MPa = 10^5 Pa

II. More on Force, Traction and Stress...

A. Forces Applied to Bodies (like rocks)

1. Body Forces: pervasive forces acting on each particle of mass throughout body of rock

2. Surface Forces: Forces applied to surfaces or contacts of the rock body

a. Forces applied per unit area

B. Surface Forces (stress or "traction")

1. Stress = Sum of all forces applied per unit area of surface

a. Normal stress component (σ_n): stress component of force applied perpendicular to the surface.

b. Shear stress component (σ_s): stress component of force applied parallel to the surface

c. Total Stress = F_{total} / Area = $\sigma_n + \sigma_s = \Sigma$

2. Newton's Second Law of Mechanical Equilibrium

a. In a body at rest, equal and opposite forces act on opposite sides of the surface such that the sum of all forces = 0

b. $F^{(\text{top of surface})} + F^{(\text{bottom of surface})} = 0$

(1) $F^{(\text{top of surface})} = -F^{(\text{bottom of surface})}$

c. Principle carries on to resulting stresses over unit areas

(1) $\sigma_n^{top} = -\sigma_n^{bottom}$

(2) $\sigma_s^{top} = -\sigma_s^{bottom}$

3. Normal Stress Relations

a. Compressive Stress: σ_n^{top} and σ_n^{bottom} are equal and opposite, pointing toward one another

(1) squeezing or compressive action

(2) **** by definition compressive stress = positive magnitude**

b. Tensile Stress: σ_n^{top} and σ_n^{bottom} are equal and opposite, pointing away from one another

(1) pull apart action

(2) **by definition tensile stress = negative magnitude**

4. Shear Stress Relations

a. Shear couple: σ_s^{top} and σ_s^{bottom} are equal and opposite.

(1) clockwise shear couple ("right lateral") = negative magnitude by definition

(2) counterclockwise shear couple ("left lateral") = positive magnitude by definition

C. Two Dimensional Stress At a Point (Stress Ellipse)

1. Identifying stress σ at a point

a. pass imaginary plane (surface) through point
b. identify normal and shear stress, σ_n and σ_s

2. 2-D Plane of Reference

a. "Stress Elipse": complete graphical representation of total stress σ at a point in space

(1) Principal Stresses

(a) Maximum and Minimum stresses acting on planes through a given point at right angles to one another in the stress elipse

(b) Maximum Stress (highest magnitude) = σ_1
(c) Minimum Stress (lowest magnitude) = σ_3

(d) By definition, σ_1 >/= σ_3

(2) Requirements of Principal Stress Field

(a) Magnitude and directions of σ_1 and σ_3 uniquely define stress field in 2D at a given point.

(b) Since σ_1 and σ_3 act perpendicular to principal surfaces, the component of shear stress in each is = 0 (i.e. σ_1 and σ_3 are comprised totally of normal components)

(c) principal planes are those which the principle stresses are acting upon

i) principal stress acts perpendicular to principal planes

(d) System must be at mechanical equilibrium, with principal stresses acting in equal and opposite manner

D. Three Dimensional Stress At a Point (Stress Ellipsoid)

1. Expand stress ellipse into third dimension = "principal stress ellipsoid"

a. 3-mutually perpendicular axes to ellipsoid

b. 3-mutually perpendicular principal stresses acting normal to principal surfaces of cube in 3-D (shear stress component of each principal stress = 0 by definition)

(1) σ_1 = maximum principal stress (long axis of magnitude on ellipsoid)
(2) σ_2 = intermediate principal stress (intermediate axis of magnitude on ellipsoid)
(3) σ_3 = minimum principal stress (short axis of magnitude on ellipsoid)

c. By definition: σ_1 >/= σ_2 >/= σ_3

d. Vectoral Components of σ_1 , σ_2 and σ_3 relative to x,y,z coordinate system

(1) σ_1 : divided into x,y,z subvectors
(2) σ_2 : divided into x,y,z subvectors
(3) σ_3 : divided into x,y,z subvectors

(a) hence nine vectoral components are needed to uniquely describe the stress exerted on a given point in 3-dimensions

III. What Is a Tensor Then You Ask???

A. Tensor: mathematical quantity used to describe the physical properites of a material

1. Defined by a set of scalar components

B. Tensor components: the no. of scalar or vectorial variables needed to describe a system completely

1. Tensor Rank: exponential value used to indicate how many components are required to describe the system

a. $c = d^r$ where c = no. of components of system; d = the dimension of the physical space (e.g. 1, 2 or 3), r = rank of tensor described by exponent

C. Examples of Tensors

1. Scalar in 3-D: 1 component, so ($1 = 3^r$) where r must = 0;

a. hence a scalar (e.g. area) is a "0 rank tensor"

2. A vector in 3-D: 3 components (e.g. F1,F2, F3), so ($3 = 3^r$) where r must = 1

a. hence a single vector (e.g. force in 3-D) is a "first rank tensor"

3. A 3-D stress field with subvectors of x,y,z for each axis of stress ellipse (σ_1 , σ_2 and σ_3) for a total of 3 x 3 = 9 components, so ($9 = 3^r$) where r must = 2

a. hence a 3-D stress field is a "second rank tensor"

IV. The Mohr Diagram

A. Mohr Diagram Defined

1. Graphical Plot of stress components on x-y cartesian coordinate system

a. Horizontal axis ("x axis") = normal stress component σ_n

(1) stress units of Pa, MPa, bars

(2) compressive normal stress = positive

(3) tensile normal stress = negative

b. Vertical axis ("y axis") = shear stress component σ_s

(1) counterclockwise shear stress (left lateral) = positive
(2) clockwise shear stress (right lateral) = negative

2. Mohr Diagrams and Experimental Rock Mechanics

a. Mohr diagram derived from lab testing of rock strength

b. Method

(1) place rock core of given diameter and length in triaxial press
(2) rock core under confining pressure from sides with $\sigma_2 = \sigma_3$
(3) Stress applied vertically in press

(a) compression: $\sigma_1 > \sigma_2 = \sigma_3 > 0$
(b) tension: $\sigma_1 = \sigma_2 > \sigma_3$; $\sigma_1, \sigma_2 > 0$; $\sigma_3 < 0$

(4) Stress applied until rock core undergoes failure

(a) confining stress recorded
(b) active stress at point of failure recorded

(5) Complete multiple runs on rock core, varying confining pressure, actively stressing until rock fails.

(a) record data

c. Plot of "Mohr Circle"

(1) Mohr Circle: a circular plot of stress applied to a given point, that defines stress components acting on all possible planes passing through that point

(a) Center of Mohr circle lies on x-axis, or normal stress axis of Mohr Diagram

(2) Principal stress components

(a) stress difference = $\sigma_1 - \sigma_3$
(b) σ_1 and σ_3 plotted as points on the x-axis of the Mohr diagram (i.e. plotted on normal stress axis)
(c) σ_1 and σ_3 define unique points that lie on the envelope of the Mohr Circle, and on the σ_n axis, where $\sigma_s = 0$.

i) hence σ_1 and σ_3 are entirely normal in composition at these unique points

(3) Surface stress and orientation of planes

(a) pass a plane in space through stress point

i) apply σ_1 to the plane

ii) resolve normal and shear stress components operating on this plane

iii) normal and shear stress components a function of angle between plane and σ_1 .

a) at 90 degrees, $\sigma_1 = \sigma_n$ and $\sigma_s = 0$.

b) at 0 degrees, $\sigma_1 = \sigma_s$, and $\sigma_n = 0$

c) between 0-90 degrees, varying components of σ_n and σ_s , accordingly

(b) Back to the Mohr circle...

i) angle theta θ = angle between σ_1 and σ_n operating on plane

ii) On mohr circle, θ is doubled and represented by 2θ

a) θ has values of 0-180 in physical space

b) 2θ has values of 0-360 in terms of generating the Mohr circle

(4) Resolving normal and shear stress components acting on a plane

(a) Mohr circle:

i) circle drawn with center lying on x-axis or σ_n axis of Mohr Diagram

ii) Diameter and position of circle defined by σ_1 and σ_3

a) σ_1 and σ_3 plotted as points on σ_n axis of Mohr diagram where $\sigma_s = 0$.

b) Mohr circle drawn through σ_1 and σ_3 , with diameter of circle = $\sigma_1 - \sigma_3$

(b) center of mohr circle on σ_n axis

i) center point = $(\sigma_1 + \sigma_3)/2$

(c) r = line of radius r drawn from center of mohr circle to outside perimeter of circle

(d) 2θ = derived from θ which is angle between σ_1 and σ_n operating on plane

i) angle 2θ formed between
   a) line r, center point of Mohr circle, and line from center point to σ_1 on x-axis

   b) 2θ ranges from 0-360 measured in a counterclockwise direction on Mohr circle

ii) All points that lie on Mohr circle define stress components for σ_n and σ_s acting on all planes that pass through a given point in space

   a) Points on Mohr circle uniquely described by components (σ_n, σ_s)

(e) Stress Component Resolution

i) Given data:

   a) plane oriented at angle θ relative to σ_1
   b) Maximum and minimum stress defined by σ_1 and σ_3 respectively

ii) Component Equations

Normal Stress Component: $\sigma_n = [(\sigma_1 + \sigma_3)/2] + ([((\sigma_1 - \sigma_3)/2) \text{Cos } 2\theta]$

Shear Stress Component $\sigma_s = [(\sigma_1 - \sigma_3)/2] \text{ Sin } 2\theta$

V. Terminology for States of Stress

A. Hydrostatic Pressure (i.e. lithostatic pressure)
1. all principal stresses are compressive and equal
2. $\sigma_1 = \sigma_2 = \sigma_3$
3. shear stress = 0
4. Plot on Mohr circle reduces to point on the normal stress axis

B. Uniaxial Stress

1. Uniaxial Compression

a. $\sigma_1 > \sigma_2 = \sigma_3 = 0$

2. Uniaxial Tension

a. $0 = \sigma_1 = \sigma_2 > \sigma_3$ (i.e. σ_3 is negative)

C. Confined Compression

1. $\sigma_1 > \sigma_2 = \sigma_3 > 0$
2. uniaxial compression + hydrostatic stress

D. Extensional Stress

1. $\sigma_1 = \sigma_2 > \sigma_3 > 0$
2. uniaxial tension + hydrostatic stress