With the example discussed in this section, we hope to show the different usages by the three methods. Before the availability of computer codes such as those in `Ch3.apr`, exploring various approaches to the analysis or detection of spatial patterns would have involved a tremendous effort. Now it is feasible and convenient.

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CHAPTER 4

LINE DESCRIPTORS

In previous chapters, we have discussed how certain types of geographic features or events can be represented abstractly by points in a GIS environment such as ArcView. We have also discussed how locational information of these point features or events can be extracted from ArcView and utilized to describe their spatial characteristics. Furthermore, we have discussed the methods used to analyze them with other attribute data describing the features. In this chapter, let's shift our attention to the description and analysis of linear geographic features that can be represented most appropriately by line objects. We will describe two general linear features that can be represented in a GIS environment. Then, as in the previous chapter, we will discuss how geographic information can be extracted to study these linear features. Most of these analyses are descriptive.

4.1 THE NATURE OF LINEAR FEATURES

In a vector GIS database, linear features are best described as line objects. As discussed in Chapter 2, the representation of geographic features by geographic objects is scale dependent. For instance, on a small-scale map (1:1,000,000), a mountain range may be represented by a line showing its approximate location, geographic extent, and orientation. When a larger scale is adopted (1:24,000) or more detailed information is shown, a line is too crude to represent a mountain range with a significant spatial extent at that scale. At that scale, a polygon object is more appropriate. In other words, a geographic feature with significant spatial extent can be represented abstractly by a linear object at one scale but by a polygon at another scale. This process is sometimes known as *cartographic ab-*

straction. Another example is the Mississippi River and its tributaries. They have significant widths when they are shown on large-scale maps, but they are represented by lines on small-scale maps.

A line can be used to represent linear geographic features of various types. Most people and even GIS users use linear features for rivers and roads, but they actually can represent many more types of geographic features. Features of the same type within the same system are generally connected to each other to form a network. For instance, segments of roads connected together form a road network, and segments of streams belonging to the same river system or drainage basin form a river network or a drainage network of their own. Within these networks, individual line segments have their own properties. For example, each of them has a different length, different beginning and ending points, or different water/traffic flows. They are related to each other in a topological manner. These segments cannot be treated separately because of this topological relationship. Other examples include the network of power utility lines and the pipeline system of a gas utility.

Linear features, however, do not have to be connected to each other to form a network. Each of these features can be interpreted alone. For instance, fault lines of a geologically active area are probably noncontiguous. Other examples include spatially extensive features such as mountain ranges or touchdown paths of tornados. These spatially noncontiguous linear features can be analyzed as individual objects without any topological relationships.

Line objects in a GIS environment are not limited to representing linear geographical features (either networked or nonnetworked). They can also be used to represent phenomena or events that have beginning locations (points) and ending locations (points). For instance, it is quite common to use lines with arrows to show wind directions and magnitudes, which are indicated by the lengths of the lines. These are sometimes referred to as *trajectories*. Another example is tracking the movements of wild animals with Global Positioning System (GPS) receivers attached to them over a certain time period. In that case, the line objects represent where they started and where they stopped.

Figures 4.1–4.3 are examples of these three types of linear objects in GIS. Figure 4.1 shows a set of fault lines in Loudoun County, Virginia. Some of these fault lines are joined together for geological reasons. But topologically, there is no reason for these fault lines to be linked. In fact, Figure 4.1 shows many fault lines separately from other fault lines. In contrast to fault lines, which are geographic features, the linear objects shown in Figure 4.2 are events. The lines in Figure 4.2 show the trajectories of wind direction and speed (they can also be thought of as magnitude) at given locations. There are not geographic features we can observe, but there are geographic phenomena they can be represented. The line objects in Figure 4.2 represent the two common attributes of linear geographic features: direction and length. Different lines can have similar orientations (for instance, from north-northeast to south-southwest in Figure 4.1), but the directions of these lines can be opposite to each other. Therefore, an arrow is added to each line to show the direction of the wind. The length of the line can represent the spatial

Figure 4.1 Selected fault lines in Loudoun County, Virginia.

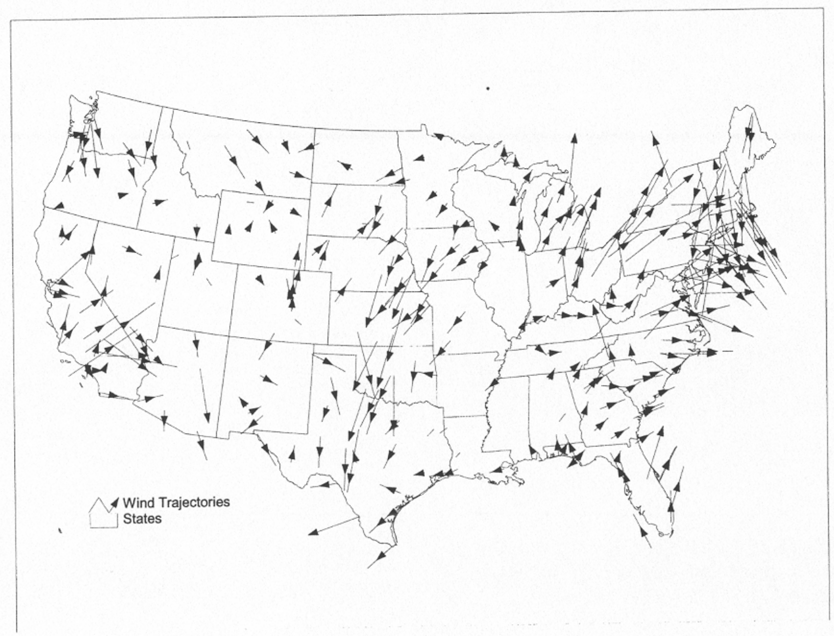


Figure 4.2 Trajectories of wind for selected U.S. locations.

Figure 4.3 Major road networks in the United States.

extent of the linear object, but in Figure 4.2, the length is proportional to the strength or magnitude of the wind. Figure 4.3 is a standard road network with line segments linking to each other. In addition to the characteristics of the other two types of linear features, a network of features shows how individual linear features are related topologically.

4.2 CHARACTERISTICS AND ATTRIBUTES OF LINEAR FEATURES

4.2.1 Geometric Characteristics of Linear Features

In most vector-based GIS, a linear object is defined either by a line segment or by a sequence of line segments that is sometimes referred to as a *chain*. If the object is relatively simple, such as the short, small fault lines shown in Figure 4.1, then a simple line segment will be adequate. But if the object is one side of the curb of a winding street, then several line segments may be required to depict its nonlinear nature. Similarly, a chain instead of a simple line will be rather effective and accurate in depicting an interstate highway.

If a line is used to represent a simple linear geographic feature, we would need two endpoints to define it. The locations of these two points may be defined in the form of longitude-latitude, x-y, or another coordinate system. Using the fault line as an example again, if we know that the fault line is short and a simple line

segment is adequate to represent it, then all we need are the locations of the two points at both ends of the line. If a chain is required to depict a more complicated linear feature, in addition to the two endpoints of the chain, intermediate points depicting the sequence of line segments are needed to define the linear feature. The curb of a street is basically a collection of the edges of concrete blocks. Therefore, a line segment defined by two endpoints can represent the edge of a block, and a sequence of segments defined by two terminal points and a set of intermediate points can represent the curb. These chains can exist without linking to each other, like the fault lines in Figure 4.1. If chains are connected to each other, they form a *network*. In a network, linear objects are linked at the terminal points of the chains. A terminal point of a chain can be the terminal point of multiple chains, such as the center of a roundabout in a local street network or a major city such as St. Louis, where several interstate highways (I-70, I-55, I-64 and I-44) converge.

4.2.2 Spatial Attributes of Linear Features: Length

Linear features in GIS can carry attributes just like other types of feature. Here we focus only on spatial attributes that can be derived from linear features. To simplify the discussion, we can treat a simple line and a chain in the same manner in the sense that they can generally be defined by two terminal locations. In fact, most GIS made the intermediate points transparent to users. An obvious spatial attribute of any linear feature is its length. Given the locations of the two endpoints, the length of the linear feature can easily be calculated. After extracting the location of the two endpoints, we can apply the Pythagorean theorem to calculate the distance between the points and thus the length of the linear feature. The Pythagorean theorem states that for a right angle triangle (Figure 4.4), the sum of the squares of the two sides forming the right angle is equal to the square of the longest side. According to Figure 4.4, $a^2 + b^2 = c^2$. Therefore, if we know the x-y coordinates of the endpoints of c, then using the theorem, the length is

$$c = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

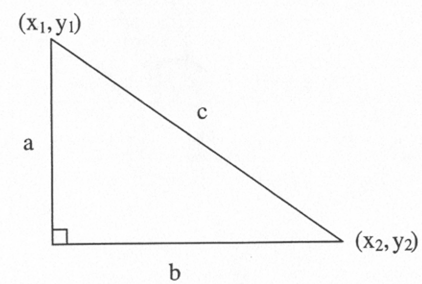


Figure 4.4 The Pythagorean theorem.

The above equation is appropriate to calculate the length of a simple line segment. If a set of line segments is linked together to form a chain, the length of the chain can be defined as the sum of these individual line segments if we are interested in the aggregated length of the chain. But if we are interested only in the spatial extent of the linear geographic feature, such as how far a river, which may be meandering, extends inland, then we can just calculate the straight-line length of the chain. This can be accomplished by taking the terminal locations of the chain and deriving the length between them as if a straight line is formed using the two terminal locations.

ArcView Notes In the ArcView GIS environment, linear features are represented by the shape polyline. A polyline can be as simple as a simple line segment with only two endpoints. It can be more complicated, like a chain with multiple line segments concatenated together to depict more complicated geographic features. ArcView does support the simple line shape or object. But when geographic objects are stored in shapefiles, only the polyline shape is recognized.

Some spatial data sets used by ArcView may have length as an existing attribute. Otherwise, the information on length has to be extracted. Fortunately, in ArcView, we do not have to extract the locations of the endpoints and then apply the Pythagorean theorem in order to obtain the length.

The length of polyline shapes can be extracted either by using the **Field Calculator** with `.ReturnLength` request issued to the Polyline shape objects (please refer to Chapter 2) or by selecting the menu item **Add Length and Angle** in project file `Ch4.apr`. Either approach will put the length into the attribute table associated with the polyline theme in ArcView.

To use the **Field Calculator**, first choose **Start Editing** on the table under the **Table** menu. Under **Edit**, choose **Add Field** to add a field for the length. Please note that the `.ReturnLength` request provides the actual or true length of the polyline. Then use the **Field Calculator** to compute the length by inserting `[shape].ReturnLength` into the window.

If the polyline feature does not follow a straight line, the request will return the length following all the line segments when they sway from side to side. If the user is interested only in the length of the polyline feature marked by the two endpoints (straight line length), then the request `.AsLine.ReturnLength` should be used instead. This request first converts the polyline into a simple line segment defined by the two endpoints of the chain and calculates the length based upon the simplified line segment.

Another method used to extract and add the length information to the ArcView feature table is to use the new menu item created in the project file `Ch4.apr`. The interface in this project file is shown in Figure 4.5. Under the **Analysis** menu, a new item is **Add Length and Angle**. Choose this menu item to extract and add the length information of each polyline object to the feature table. The user will be asked to choose the straight-line length or the true length. Figure 4.6 shows part of the feature table after the two types of length information are extracted and added. The length attribute in the second column was originally provided in the table. Since the menu item **Add Length and Angle** was chosen twice, but the straight line length the first time and the true length the second time, both types of length were added to the table. It is clear that the first polyline feature, a long fault line in Figure 4.1, is a chain with segments going left and right. Thus, the *SLength* (for straight line length) is shorter than the *Tlength* (true length). The second polyline feature, however, is a rather short fault represented by a simple line segment. Therefore, *SLength* and *Tlength* are the same. After this step, the user can use the ArcView Statistics function to obtain simple descriptive statistics.

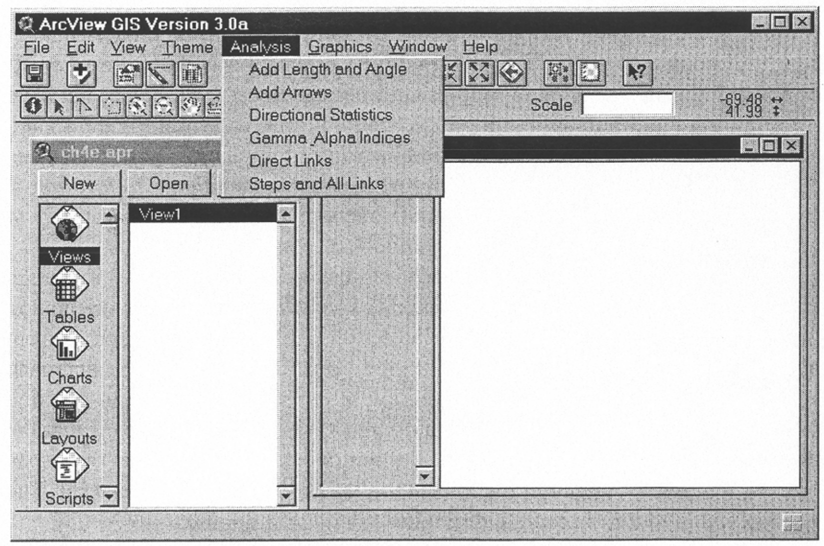
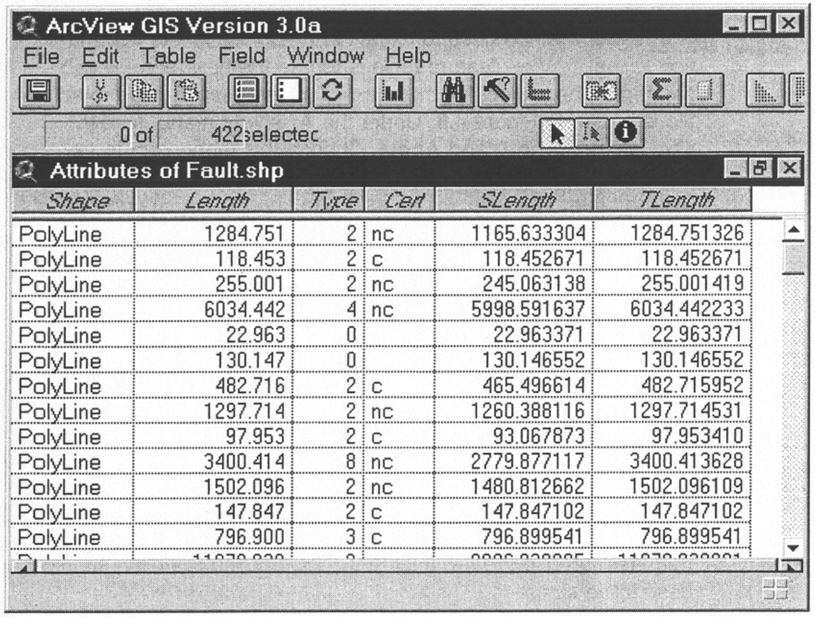


Figure 4.5 User interface of `Ch4.apr`.



Shape	Length	Type	Cert	SLength	TLength
PolyLine	1284.751	2	nc	1165.633304	1284.751326
PolyLine	118.453	2	c	118.452671	118.452671
PolyLine	255.001	2	nc	245.063138	255.001419
PolyLine	6034.442	4	nc	5998.591637	6034.442233
PolyLine	22.963	0		22.963371	22.963371
PolyLine	130.147	0		130.146552	130.146552
PolyLine	482.716	2	c	465.496614	482.715952
PolyLine	1297.714	2	nc	1260.388116	1297.714531
PolyLine	97.953	2	c	93.067873	97.953410
PolyLine	3400.414	8	nc	2779.877117	3400.413628
PolyLine	1502.096	2	nc	1480.812662	1502.096109
PolyLine	147.847	2	c	147.847102	147.847102
PolyLine	796.900	3	c	796.899541	796.899541

Figure 4.6 Portion of feature table after length information is added.

4.2.3 Spatial Attributes of Linear Features: Orientation and Direction

Another obvious spatial attribute of a linear feature is its orientation. Orientation here is nondirectional. For instance, east-west orientation is the same as west-east orientation. Orientation is appropriate when the linear feature does not have a directional characteristic. For instance, the fault lines shown in Figure 4.1 are nondirectional. There is no specific *from-to* nature for each of those fault lines even though we can describe the fault line using *from location x to location y*. But if the fault line has orientation but not direction, then using *from* location *y to* location *x* to describe the fault line does not change its nature.

Other examples of nondirectional linear features include curbs, mountain ranges in small-scale displays, and sections of coastline. Usually, the orientation of a set of linear features can be indicated by verbal description, such as from north to south or from east to west, or vice versa. Another common approach is to use an angle, measured counterclockwise in reference to east, to describe the orientation precisely. Therefore, an orientation of 45 degrees means that the overall trend of the linear features is 45 degrees counterclockwise from the *x*-axis or east. Sometimes, however, the orientation may refer to the north instead. The referencing direction is situation dependent.

Arguing that direction is not appropriate for some linear geographic features does not imply that GIS data used to represent these nondirectional features do

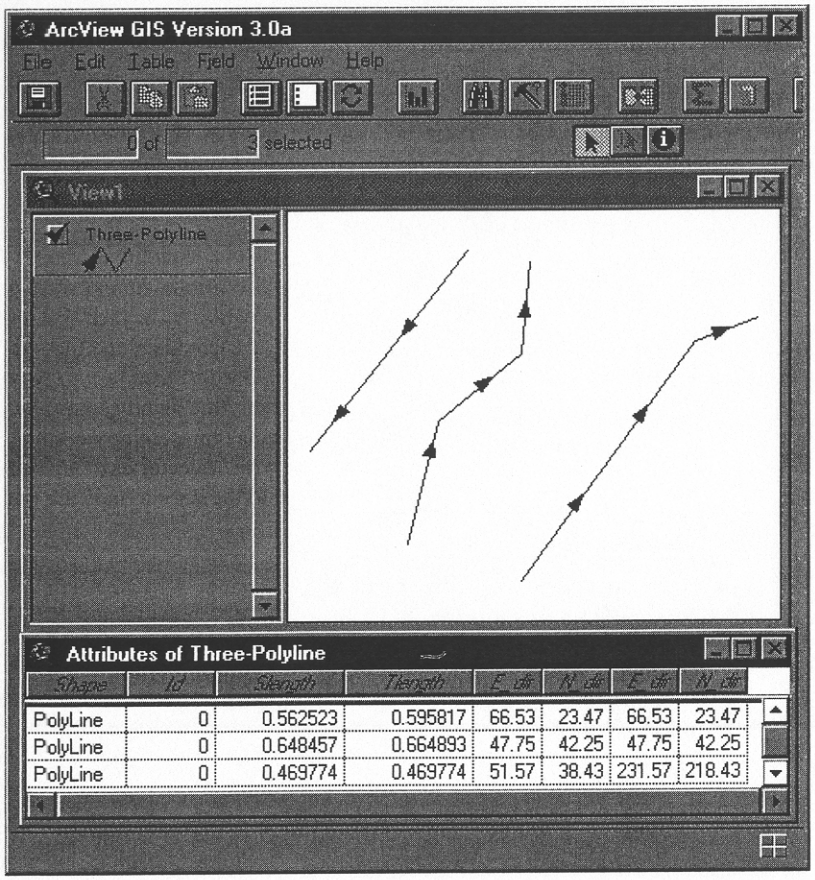
not record the directional information. In fact, most GIS capture the directions of linear features as the data are created even if the directions are not meaningful to the features. Depending on how the data are entered into GIS, quite often the beginning point and the ending point of a chain during the digitizing process define the direction of the chain. Therefore, the directions of fault lines, for example, are stored in the GIS data when the data are created even though direction is inappropriate to describe the fault lines.

With respect to orientation, a similar attribute of linear features is their direction. As implied in the discussion above, linear features have directional characteristics that are dependent on the beginning and ending locations. *From* location *x to* location *y* is not the same as *from* location *y to* location *x*. In fact, the directions of the two descriptions are exactly the reverse of each other, and the two descriptions can refer to two different linear features. For instance, a two-way street can be described as two linear features with exactly the same geographic location and extent but opposite directions. Linear objects representing events or spatial phenomena are often directional in nature. The wind trajectories described in Figure 4.2 are clearly of this type. Arrows are added to the lines to indicate the directions.

ArcView Notes In the ArcView GIS environment, it is not too easy to extract orientation or direction information from linear features, not quite similar to the extraction of length information. There is no request in ArcView to obtain the angle of a polyline. Based on the locations of the two terminal points, the orientation or angle of a linear feature can be obtained using trigonometric functions. In practice, this rather tedious process could be implemented using the **Calculate** function under the **Table** menu item. To simplify the procedure, we have developed a new menu item, **Add Length and Angle**, in the project file `Ch4.apr` to accomplish this task.

When that menu item is selected, users will be asked if they would like to add the directional information to the feature table. Then users will be asked if they would like to use orientation or direction. Users also have the choices of referencing to the east or the north in deriving the orientation or angle of direction. In order to demonstrate clearly the difference between orientation and direction, a simple make-up data set is used.

Figure 4.7 shows a simple three-polyline example. Arrows are added to the polylines to show their directions. Obviously, the three polylines have similar orientations. Two of them, however, have similar directions and the third has an almost opposite direction. Then **Add Length and Angle** was run four times to exhaust all four possibilities: orientation referenced to the east, orientation referenced to the north, direction referenced to the east,



Shape	Id	Slength	Tlength	E_dir	N_dir	E_dir	N_dir
PolyLine	0	0.562523	0.595817	66.53	23.47	66.53	23.47
PolyLine	0	0.648457	0.664893	47.75	42.25	47.75	42.25
PolyLine	0	0.469774	0.469774	51.57	38.43	231.57	218.43

Figure 4.7 Hypothetical polylines and resultant feature tables.

and direction referenced to the north. The resultant feature table is also shown in Figure 4.7. These results confirm our visual inspections.

In the feature attribute table shown in Figure 4.7, the third and fourth columns present the length information. The fifth and sixth columns show orientation referencing to the east and north, and the seventh and eighth columns show angles of direction referencing to the east and north. The table clearly shows that the three polylines have similar orientations (between 48 and 67 degrees when referenced to the east). In regard to direction, the

first two polylines have similar angles of direction (48 and 67 degrees when referenced to the east) but the third polyline has an opposite direction (232 degrees). Please note that for both orientation and direction, the angles are measured *counterclockwise* from the east and *clockwise* from the north.

4.2.4 Spatial Attributes of Linear Features: Topology

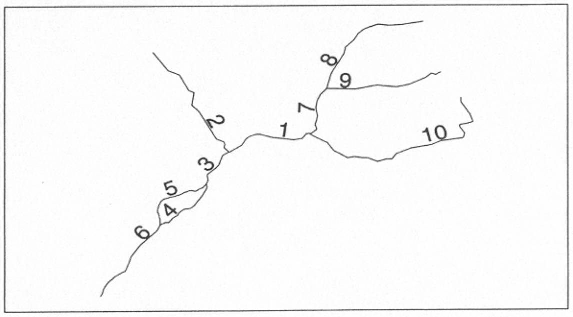
In a network database, linear features are linked together topologically. The attributes described in previous sections, such as length or spatial extent and orientation or direction, are also applicable to segments of the network. The length of the network, which can be defined as the aggregated lengths of individual segments or links, is an important feature in analyzing a network. Orientation or direction, depending on the specific natural of the network, is essential in understanding the geographic setting of the network. For instance, the direction of flow in tributaries of a river network should yield a consistent direction if the watershed is not very large.

The orientation of a major highway, to some extent, may partially reflect some characteristics of the landscape. Clearly, at a local scale, the direction of a local street network is important in planning the traffic pattern. All the concepts and analyses of linear features discussed so far are applicable in analyzing segments or links in a network. An additional aspect in a network, however, is how different segments are linked together and how these segments are related to each other. This is part of the general topic of the topological structure of a network.

The most essential topological aspect or attribute of a network is how different links or edges are connected to each other. This is sometimes known as the *connectivity* of a network. To capture quantitatively how different links are joined to other links, a traditional method is to use a matrix to store and represent the information. Assume that we have n links or edges in a network and each link has a unique identifier (ID). Conventionally, the labels of the columns are the ID numbers, and so are the labels of the rows in the *connectivity matrix*. The matrix is a square, that is, the number of rows equals the number of columns. A cell in the matrix captures the topological relationship between the two links denoted by the IDs in the corresponding row label and column label. If the two links are directly joined to each other, the cell will have a value of 1. Otherwise, the value will be 0.

Sometimes a link or an edge is not connected to itself, and therefore all diagonal elements in the matrix are 0s. Because each cell carries a value of either 0 or 1, this type of matrix is also called a *binary matrix*. And because the relationship between any pair of edges is symmetrical—that is, if Link A is connected to Link B, then Link B is also connected to Link A—the matrix is symmetrical. The triangle of matrix in the upper right side of the diagonal is a mirror of the lower left triangle.

Figure 4.8 shows a subset of major roads in the northeastern United States. All these roads in this example are in the state of Maine. Each road segment is la-



ID	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	1	0	0	1
2	1	0	1	0	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0
4	0	0	1	0	1	1	0	0	0	0
5	0	0	1	1	0	1	0	0	0	0
6	0	0	0	1	1	0	0	0	0	0
7	1	0	0	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1	0	1	0
9	0	0	0	0	0	0	1	1	0	0
10	1	0	0	0	0	0	1	0	0	0

Figure 4.8 A road network and its connectivity matrix.

beled with a unique ID. There are 10 road segments in this system. To capture the topological relationship among these segments, a connectivity matrix is created. Figure 4.8 also includes the connectivity matrix of the road network. The row labels and column labels refer to the IDs of the links or segments in the network. All diagonal elements, indicating whether each segment is connected to itself, are 0s.

In Figure 4.8, road segment 1 is in the middle of the network. It is connected to segment 2 to the northwest, segment 3 to the west, segment 7 to the northeast, and segment 10 to the east. Therefore, in the first row of the matrix, which indicates what road segments segment 1 is connected to, 1s are found in the cells for segments 2, 3, 7, and 10. The same principle is applied to other rows in the matrix. But if we focus on the first column, the information captured there is the same as that in the first row. Therefore, this is a symmetrical matrix.

Connectivity of links is the most fundamental attribute of a network. Any analysis of a network has to rely almost exclusively on the connectivity attribute. In fact, one may argue that connectivity defines a network. For instance, a set of linear features is shown on the screen of a GIS package. If we want to find the total length of that network, we may just add up the lengths of the individual segments appearing on the screen. But with very detailed visual inspection, we may find that two of those links that appeared to be joined together are in fact separated. Therefore, without topological information, any analysis performed on a network

could be erroneous. In later sections, we will show how topological information serves as the basis of a suite of tools for network analysis.

We have briefly discussed attributes of linear features that are not necessarily connected and attributes of linear features that are connected topologically to form a network. To summarize, attributes that are applicable to linear features, but dependent on the nature and characteristics of these features, include length, orientation, direction, and connectivity. Please note that not all of these attributes are present or appropriate for all types of linear features. In the next section, we will discuss how different attributes of spatially noncontiguous linear features can be used to support various analytical tools specifically designed for these features. The following section will discuss tools we can use to analyze linear features in a network.

ArcView Notes In the project file for this chapter (`Ch4.apr`), a new menu item, **Connectivity Matrix**, is added for users to construct the connectivity matrix of a given network, as shown in Figure 4.8. The connectivity matrix will be a binary symmetric matrix in `dbf` format.

In some network or polyline databases, there may not be a unique identifier for each polyline in the ArcView feature table. The Connectivity Matrix function in the project file will ask the user if this situation exists. If there is no attribute item in the table that can be used as the unique identifier for each polyline, the new function will create a sequential unique identification number for each polyline to be used as the row and column labels in the connectivity matrix.

In other words, if there is no unique identifier in the feature table, but the user chooses one of the attributes as the identifier, the output may not be able to show the topological relationship among polylines accurately, as there is no way to distinguish one polyline from the others based upon the column or row labels.

In ArcView, any two objects will have a distance of 0 if any parts of the two objects touch each other. Using this characteristic in defining distance, the Avenue script for the connectivity matrix requests the distance between all pairs of polylines. If the distance between a given pair of polylines is 0, this means that they are connected; thus, a 1 will enter the corresponding cell in the matrix. Please note that we assume that the network structure follows the planar graph principle. That is, when two lines cross each other, a new vertex will be created; thus, the two lines are broken into four lines or segments. We do recognize, however, that two lines could cross each other without creating an intersection. For instance, a skyway over another highway or an overpass will be shown as two lines crossing each other, but in fact, topologically they do not intersect.

4.3 DIRECTIONAL STATISTICS

To analyze linear geographic features in greater depth, we have to rely on statistical techniques specifically designed for linear features. Unfortunately, not many statistical tools are available for analyzing linear features. Most of these techniques can be considered geostatistics developed and used mostly by geoscientists (Swan and Sandilands, 1995). Before we discuss these techniques, some preliminary and exploratory analyses can be conducted.

4.3.1 Exploring Statistics for Linear Features

In Section 4.1, we discussed the process used to extract some basic statistics or attributes of linear objects and to store the information in the feature attribute table as additional attributes. These statistics offer opportunities to conduct some preliminary and exploratory analyses. The length of a linear object—both the straight-line length and the truth length—can be analyzed using standard descriptive statistics such as mean, variance, and so on. In most situations, the analyses based upon these two length measures will probably yield slightly different results; however, the differences should not be dramatic. Table 4.1 shows two statistical summary tables of the fault line coverage shown in Figure 4.1. Part a

TABLE 4.1 Statistics Describing the Lengths of Fault Lines

(a) Statistics for the Slength Field	
Sum:	932,332.644656
Count:	422
Mean:	2,209.319063
Maximum:	39,005.08889
Minimum:	19.876572
Range:	38,985.212318
Variance:	16,315,314.639887
Standard deviation:	4,039.222034

(b) Statistics for the Tlength Field	
Sum:	648,470.093148
Count:	422
Mean:	2,247.559462
Maximum:	41,313.702596
Minimum:	19.876572
Range:	41,293.826024
Variance:	17,342,308.454853
Standard deviation:	4,164.409737

summarizes the attribute SLength, and Part b summarizes the attribute TLength. Conceptually, the two attributes should have the same value for a linear feature if the feature is represented by a simple line segment but not a chain. When a chain is needed, the straight-line length (SLength) will be shorter than the length of the entire chain (TLength). The two sets of summary statistics, including mean, sum, variance, and standard deviation, reflect these nature of the two attributes.

To go one step further, based on the difference between the straight-line length and the full length of the chain, we can analyze the topological complexity of each linear feature in the data set. One simple method is to derive a ratio of the two length measures—sinuosity (DeMers, 2000). When the length of the entire chain is divided by the length of the straight-line distance, the ratio is 1 if the linear feature is simple enough to be represented by a simple line segment. The higher this ratio is, the more complex the linear feature is. Table 4.2 shows selected fault lines with ratios larger than 1.2 in descending order. As a result, 10 fault lines have a ratio larger than 1.2. The first eight of them are shown in the top panel in Figure 4.9. All these faults are short but banned at one end, creating relatively large ratios for chain length to straight-line length. The other two fault lines on the list are shown in the middle panel of Figure 4.9. The ninth fault line looks like a curve, while the tenth is crooked. The ratio is very effective in identifying linear features that are relatively crooked.

Another attribute of a linear feature is direction. A simple exploratory method used to study linear features with a directional attribute is to add arrows to those features in a display to provide visual recognition of the pattern, if any. Figure 4.2, showing the trajectories of wind, provides a useful visual display of the phenomenon. Based upon the wind directions displayed in Figure 4.2, we can identify several circulation subsystems in the continental United States at that time. For instance, the New England region seemed to be influenced by one subsystem, while the Mid-Atlantic region throughout the South along the Atlantic coast seemed to be affected by another subsystem.

TABLE 4.2 Attribute Table for Selected Fault Lines with Length Ratios

Number	Length	Slength	Tlength	Length Ratio
1	596.841	386.532340	596.841079	1.54409
2	595.518	386.581012	595.517605	1.54047
3	698.879	483.387239	698.879055	1.44580
4	699.588	484.230785	699.558479	1.44580
5	651.754	469.902068	651.754268	1.38700
6	647.195	468.328944	647.194827	1.38192
7	603.488	441.679554	603.488095	1.36635
8	604.233	443.694169	604.232631	1.36182
9	3,400.414	2,779.877117	3,400.413628	1.22322
10	217.739	180.057803	217.738803	1.20927

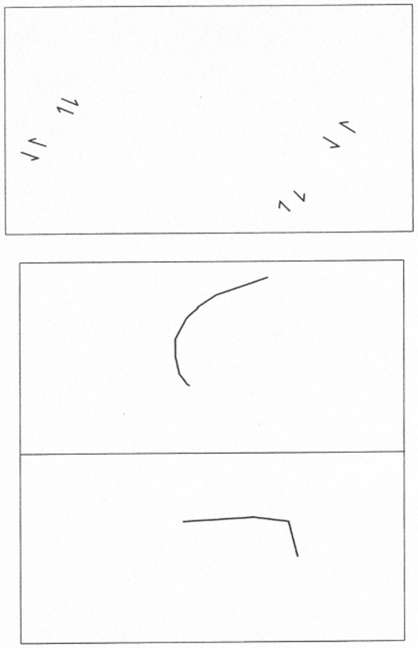


Figure 4.9 Selected fault lines with large true length to straight-line length ratio.

ArcView Notes

In the `Ch4.apr` project file, a new menu item, **Add Arrows**, is added. With an active polyline theme in the **View** document, the **Add Arrows** menu item will add arrows to individual linear features to indicate their direction. Please note the following:

1. The arrows may be small, depending on the scale of the display in reference to the size of the features. Therefore, sometimes it is necessary to zoom in in order to see the arrows. An alternative method is to choose a larger point size for the arrows with the legend editor.
2. The direction of a linear feature is defined by the sequence of the points that are entered to define the feature if the data were created by digitizing. The direction of linear features, however, can be changed using requests in ArcView.
3. Arrows can be added to any linear features, but they may not be meaningful if the linear feature has an orientation attribute but not a directional attribute. As with fault lines in Figure 4.1, adding arrows to the feature is not meaningful.

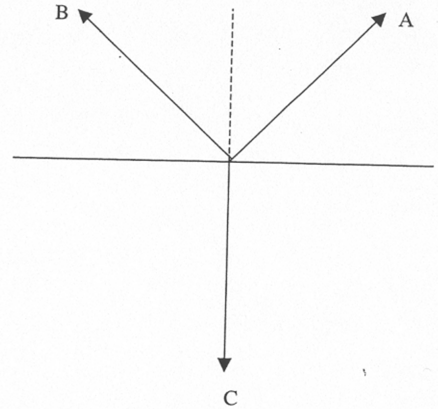


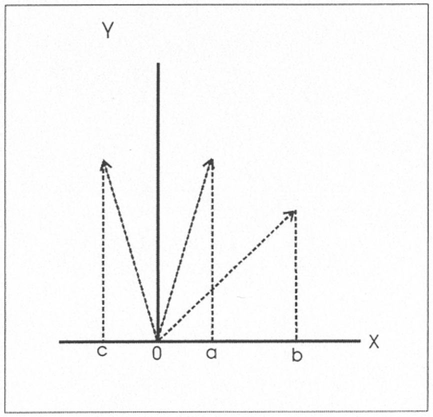
Figure 4.10 Inappropriateness of classic descriptive statistics for angle measures.

Since it is easy to analyze the length attribute of linear features, perhaps we can analyze the directional or orientation aspect of linear features with simple descriptive statistics. With the orientation and/or directional information of linear features extracted and added to the attribute data, we would naturally try to calculate descriptive statistics based on the angles (orientation or direction) of linear features. Unfortunately, descriptive classic statistics, such as mean or variance, are in general not appropriate to analyze angles of linear features. Figure 4.10 provides a simple example to illustrate this problem.

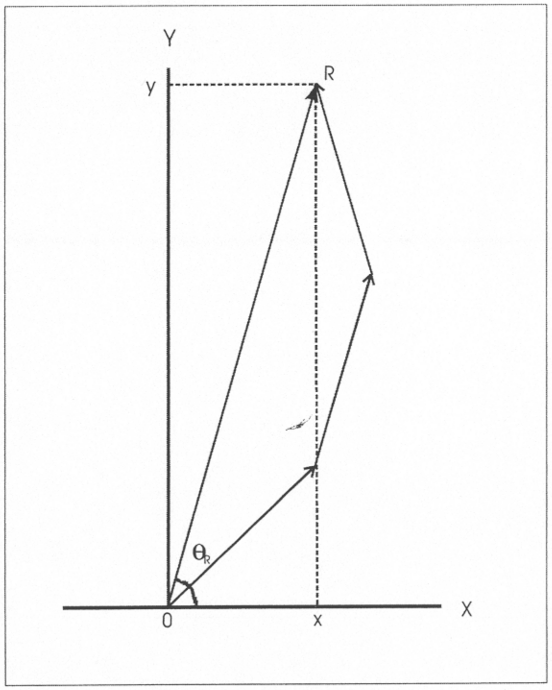
Figure 4.10 shows two vectors, A and B, with 45 degrees and 315 degrees, respectively, clockwise from the north. If we use the concept of mean in classical statistics to indicate the average direction of these two vectors, the mean of the two angles is 180 degrees, i.e., pointing south, as shown by vector **C**. But graphically, given the directions of vectors **A** and **B**, the average direction should be 0 degree (i.e., pointing north). Therefore, using classical statistical measures may be inappropriate. Because the concept of the arithmetic mean of the two angles cannot reflect the average direction, other measures such as variance cannot be defined meaningfully. To analyze angle information, we have to rely on directional statistics specifically designed to analyze vectors.

4.3.2 Directional Mean

The concept of *directional mean* is similar to the concept of average in classic statistics. The directional mean should be able to show the general direction of a set of vectors. Because directional mean is concerned with the direction but not the length of vectors, vectors can be simplified to vectors 1 unit in length (unit vectors). Figure 4.11a shows three vectors of unit length originated from O. Each vector shows a direction in reference to the origin. The directional mean is defined as the direction of the vector that is formed by "adding" all the vectors together.



(a)



(b)

Figure 4.11 Concept of directional mean.

Adding any two vectors together means appending the beginning point of the second vector to the ending point of the first vector. Figure 4.11b shows how the three unit vectors in Figure 4.11a are added together. The result is the resultant vector, OR. The directional mean of the set of three vectors, in this case θ_R, is the direction of the resultant vector, OR. The direction of the resultant vector, θ_R, can be derived from the following trigonometric relation:

$$\tan\theta_R = \frac{oy}{ox}$$

where oy is the sum of the heights of the three vectors and ox is the horizontal extent of the vectors.

Because all three vectors are unit vectors, the height of a vector (in the y-axis) is basically the sine function of the angle of the vector, and the horizontal extent of a vector (x-axis, as shown in Figure 4.11a) is the cosine function of the angle of the vector. Therefore, if the three vectors are identified as a, b, and c, and their corresponding angles are θ_a, θ_b and θ_c, then

$$\tan\theta_R = \frac{\sin\theta_a + \sin\theta_b + \sin\theta_c}{\cos\theta_a + \cos\theta_b + \cos\theta_c}.$$

To generalize, assuming that there are n vectors v, and the angle of the vector v from the x-axis is θ_v, the resultant vector, OR, forms an angle, θ_R, counterclockwise from the x-axis. Because each vector is of unit length,

$$\tan\theta_R = \frac{\sum \sin\theta_v}{\sum \cos\theta_v},$$

which is the tangent of the resultant vector. In order to find the directional mean, the inverse of tan (arc tangent) has to be taken from the above equation.

The directional mean is the average direction of a set of vectors. The idea of vector addition, as shown in Figures 4.11a and 4.11b, utilizes the fact that the vector that results from adding vectors together shows the general direction of the set of vectors. If all the vectors have similar directions, after all vectors are appended to each other, the resultant vector will be pointing at somewhere among this group of vectors.

If two vectors have different directions, such as a vector of 45 degrees and a vector of 135 degrees, the resultant vector will be 90 degrees. As discussed before, just taking the average of angles of those vectors may not be appropriate. If all vectors under consideration are smaller or larger than 180 degrees, then arithmetically *averaging* the angles will yield the correct answer. However, if some angles are smaller than 180 degrees and others are larger, then the averaging method will be incorrect.

The result derived from the above equation for directional mean, however, has to be adjusted to accommodate specific situations in different quadrants, according to Table 4.3. The table shows the trigonometric results for angles in each of

TABLE 4.3 Adjusting the Directional Mean

sin + *ve* cos − *ve*	sin + *ve* cos + *ve*
sin − *ve* cos − *ve*	sin − *ve* cos + *ve*

the four quadrants. Because of these specific situations, the results from the calculation of directional mean should be adjusted accordingly:

1. If the numerator and the denominator are both positive in $\tan\theta_R$, no adjustment of the resultant angle is needed (the angle lies in the first quadrant).
2. If the numerator is positive and the denominator is negative (second quadrant), then the directional mean should be $180 - \theta_R$.
3. If both the numerator and the denominator are negative (third quadrant), then the directional mean should be $180 + \theta_R$.
4. If the numerator is negative and the denominator is positive (fourth quadrant), then the directional mean should be $360 - \theta_R$.

The directional mean can easily be computed if the angles of the linear feature are known. The first step is to compute the sine of each angle; the second step is to compute the cosine of each angle. Treating the sine and cosine results as ordinary numbers, we can calculate their sums to form the ratio in order to derive the tangent of the resultant vector. Taking the inverse of the tangent (arc tangent) on the ratio gives us the directional mean. Table 4.4 shows the major steps in deriving the directional mean of a selected numbers of fault lines described in Figure 4.1. First, the angles of the fault lines were extracted. Please note that directions of fault lines are not meaningful; their orientations are appropriate. Therefore, the direction mean is based upon orientation rather than direction.

The angles shown in Table 4.4 are orientations of fault lines from the east. First, the sines of the angles were derived, followed by their cosines. In many spreadsheet packages, trigonometric functions expect the angles expressed in radians instead of degrees. Therefore, the degrees of angles were first converted into radians before the sine and cosine functions were used. There are 422 fault lines in the data set. The sine and cosine values were summed. The ratio of the two sums is 1.7536, which is the tangent of the directional mean. Thus, taking the inverse tangent of 1.7536 gives us 60.31 degrees counterclockwise from the east.

4.3.3 Circular Variance

Analogous to classical descriptive statistics, *directional mean* reflects the *central tendency* of a set of directions. As in many cases, the central tendency, however, may not reflect the observations very efficiently. For instance, using the previous

TABLE 4.4 Directional Mean of Selected Fault Lines

Number	Length	E_DIR	sin_angle	cos_angle
1	1,284.751	68.56	0.930801	0.3655267
2	118.453	74.58	0.964003	0.2658926
3	255.001	94.09	0.997453	−0.071323
4	6,034.442	49.34	0.758589	0.651569
5	22.963	42.24	0.672238	0.7403355
6	130.147	43.6	0.68962	0.7241719
7	482.716	95.05	0.996118	−0.088025
8	1,297.714	89.74	0.99999	0.0045378
9	97.953	90.62	0.999941	−0.010821
10	3,400.414	57.28	0.841322	0.540534
⋮	⋮	⋮	⋮	⋮
414	2,976.265	58.1	0.848972	0.5284383
415	1,936.928	74.49	0.963584	0.2674066
416	10,528	74.19	0.96217	0.2724482
417	2,569.188	50.47	0.771291	0.6364822
418	167.585	70.12	0.949497	0.3400513
419	365.049	75.77	0.969317	0.245815
420	662.586	75.61	0.968627	0.2485208
421	274.119	77.3	0.975535	0.2198462
422	107.418	76.54	0.972533	0.2327665
		Sum =	301.914	172.76549

example, the directional mean of the two vectors, 45 degrees and 135 degrees, is 90 degrees. Apparently, the directional mean is not efficient in representing the two vectors pointing in very different directions. A more extreme case is one in which two vectors are of opposite direction. In this case, the directional mean will be pointing at the direction between them, but the statistic provides no information on the efficiency of the mean in representing all observations or vectors. A measure showing the variation or dispersion among the observations is necessary to supplement the directional mean statistic. In directional statistics, this measure is known as the *circular variance*. It shows the variability of the directions of the set of vectors.

If vectors with very similar directions are added (appended) together, the resultant vector will be relatively long and its length will be close to n if there are n unit vectors. On the other hand, if vectors are in opposite or very different directions, the resultant vector will be the straight line connecting the first point and the last point of the set of zigzag lines or even opposite lines. The resultant vector will be relatively short compared to n if for n vectors. Using the example in Figure 4.11, if all three vectors have very similar directions, the resultant vector, OR, should also be on top of the three vectors when they are appended to each other.

But in our example, the three vectors do vary in direction. Thus, OR deviates substantially from the graphical addition of the three vectors. As a result, OR is shorter than the actual length of the three vectors (which is 3). The length of the resultant vector can be used as a statistic to reflect the variability of the set of vectors. Using the same notations as above, the length of the resultant vector is

$$\mathrm{OR} = \sqrt{\left(\sum \sin\theta_v\right)^2 + \left(\sum \cos\theta_v\right)^2}.$$

Circular variance, S_v, is $1 - \mathrm{OR}/n$, where n is the number of vectors. S_v ranges from 0 to 1. When $S_v = 0$, OR equals n; therefore, all vectors have the same direction. When $S_v = 1$, OR is of length 0 when all vectors are of opposite directions, and the resultant vector is a point at the origin. Please note that the concept of circular variance is the same as the concept used when we compared the straight-line length of a chain with the length of the entire chain.

Using the fault lines as an example again,

$$\sum \sin 2_v = 301.914$$

$$\sum \cos 2_v = 172.765$$

$$\left(\sum \sin 2_v\right)^2 = 91{,}152.0634$$

$$\left(\sum \cos 2_v\right)^2 = 29{,}847.7452$$

$$\mathrm{OR} = \sqrt{(91{,}152.0634 + 29{,}847.7452)} = 347.85$$

$$S_v = 1 - (347.85/422) = 0.1757.$$

Because circular variance ranges from 0 to 1, this magnitude of circular variance is rather small, indicating that most of the fault lines have similar directions.

In the next section, we will discuss techniques commonly used in analyzing a network.

ArcView Notes

In the `Ch4.apr` file, a new menu item, **Directional Statistics**, is already in place. In constrast to previous functions that either change or add information to the feature table (such as **Add Length and Angle**) or change the View (such as **Add Arrows**), this function alters neither the table nor the view. Instead, it provides information directly to the screen.

The procedure first asks users whether orientation or direction should be used. If orientation is chosen, then polylines are treated as lines in ArcView, and polyline directions will be either from lower left to upper right or from lower right to upper left. If direction is important, the directions inherited in the polylines

will be used. The procedure will then derive the directional mean. Then the circular variance is calculated.

Using the wind trajectories in Figure 4.2 as an example, the **Directional Statistics** function is used. Because the trajectories are directional, when the **Directional Statistics** function was executed, direction was selected as important. The directional mean (counterclockwise from the east) for all the vectors is 356.54 degrees, which is an east-southeast direction. Figure 4.2 shows that there were at three major wind subsystems in the continental United States at that time. We can divide the entire region into three subregions: east, central, and west. The overall wind direction in the east is apparently pointing toward the east or slightly toward the northeast. Wind in the central area was blowing strongly toward the south or southwest. The west is a bit chaotic, but in general the directions seem to be pointing toward the southeast. Before the **Directional Statistics** function was executed, vectors in each of these three regions were selected. The results reported in Figure 4.12 confirm our speculations.

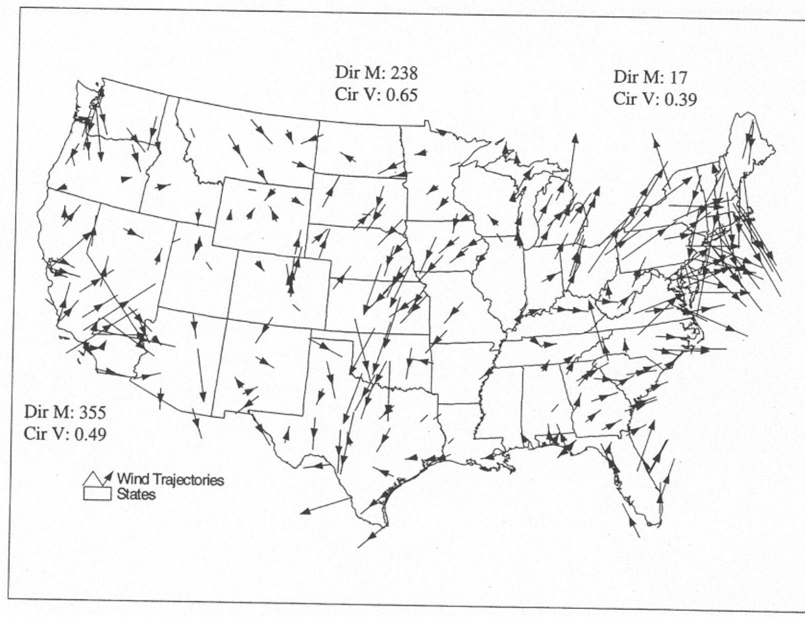


Figure 4.12 Directional means and circular variance of the three subregions.

The results of circular variance (Figure 4.12) provide additional information on wind direction in the three subregions. According to the circular variance results, the eastern section has a rather uniform direction, while the central region has the most varied wind direction. The variation of wind directions in the west is between those of the east and the south. Therefore, the spatial selection tools in GIS can be combined with the **Directional Statistics** function to analyze regional variations.

4.4 NETWORK ANALYSIS

To conduct a full range of network analyses, a special computation environment is required. The current version of ArcView cannot support this environment without extensive programming development. In addition, the Environmental System Research Institute (ESRI) has provided Network Analyst, an extension for ArcView, to support certain types of network analysis. Therefore, we will discuss topics in network analysis that will not significantly overlap with the literature relying on the Network Analyst extension.

We have already discussed the basic attribute of a network—*connectivity*. Connectivity information can be captured in a binary symmetric matrix, indicating the pairs of links or segments that are joined together. This information will be used in almost all analyses involving a network (Taaffe, Gauthier, and O'Kelly, 1996). In general, we can classify different types of network analytical tools into two categories:

- The type that accesses the overall characteristics of the entire network
- The type that describes how one network segment is related to other segments or the entire network system

The first type will be discussed in the next section on connectivity; the second type, accessibility, will be covered in Section 4.4.2.

Before we start, we have to define several terms that are used in network analysis. In network analysis, a segment of linear feature is called a *link* or an *edge*. An edge is defined by the two vertices or nodes at both ends of the edge in the network. The number of edges and vertices are often used to derive statistics indicating the characteristics of the network.

4.4.1 Connectivity

So far, we have discussed only how the connectivity information is captured and represented in the connectivity matrix. We have not analyzed the level of connectivity in a network. For a fixed set of vertices, different networks can be created

if the vertices are connected differently. When the number of vertices is fixed, networks with more edges are better connected. There is a minimum number of edges that is required to connect all the vertices to form a network.

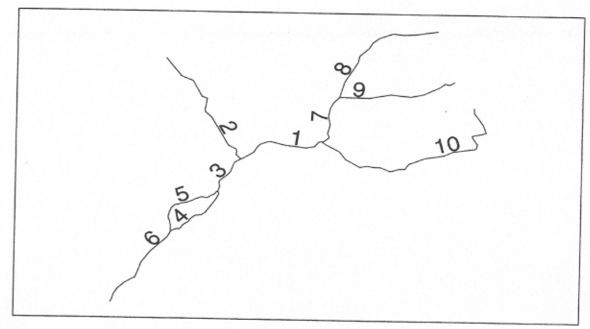
If v denotes the number of vertices and e denotes the number of edges in the network, then the minimum number of edges required to link all these vertices to form a network is

$$e_{\min} = v - 1.$$

In a *minimally connected network*, if any one edge is removed from the system, the network will be broken up into two unconnected subnetworks. In the simple network shown in Figure 4.13, there are 10 vertices and 10 edges. Since the number of vertices is 10, a minimally connected network should have 9 (10 − 1) edges. In this case, however, there are 10 edges. Therefore, the network in Figure 4.13 is not a minimally connected network. If either edge 4 or 5 is removed, then this is a minimally connected network.

Similarly, given a fixed number of vertices, there is a maximum number of edges one can construct to link all the vertices together. The edges in the maximally connected network do not cross or intersect each other (the *planar graph topology*). The maximum number of edges possible in the network is

$$e_{\max} = 3(v - 2).$$



ID	1	2	3	4	5	6	7	8	9	10
1	0	1	1	0	0	0	1	0	0	1
2	1	0	1	0	0	0	0	0	0	0
3	1	1	0	1	1	0	0	0	0	0
4	0	0	1	0	1	1	0	0	0	0
5	0	0	1	1	0	1	0	0	0	0
6	0	0	0	1	1	0	0	0	0	0
7	1	0	0	0	0	0	0	1	1	1
8	0	0	0	0	0	0	1	0	1	0
9	0	0	0	0	0	0	1	1	0	0
10	1	0	0	0	0	0	1	0	0	0

Figure 4.13 A simple network in Maine.

Therefore, the network in Figure 4.13 can have up to 24 edges ($3 * (10 - 2)$) for the given number of vertices in the network. But actually, the network has only 10 edges. Thus, the Gamma Index (γ), which is defined as the ratio of the actual number of edges to the maximum possible number of edges in the network, is

$$\gamma = \frac{e}{e_{\max}}.$$

The Gamma Index for the network in Figure 4.13 is 10/24, which is 0.4167. The Gamma Index is most useful in comparing different networks to differentiate their levels of connectivity. To illustrate this aspect of the index, another network selected from the highway network in northern New Jersey is shown in Figure 4.14. It is easy to tell by visual inspection that the New Jersey network is much better connected than the highway network in Maine. To verify this conclusion, we calculate the Gamma Index. In the New Jersey network, there are 17 vertices and 21 edges. Therefore, the Gamma Index is 0.4667, higher than that in the Maine network.

Another characteristic of connectivity is reflected by the number of circuits that a network can support. A *circuit* is defined as a closed loop along the network. In a circuit, the beginning node of the loop is also the ending node of the loop. The existence of a circuit in a network implies that if it is for a road network, travelers can use alternative routes to commute between any two locations in the network. A minimally connected network, as discussed above, barely links all vertices together. There is no circuit in such a network. But if an additional edge is added

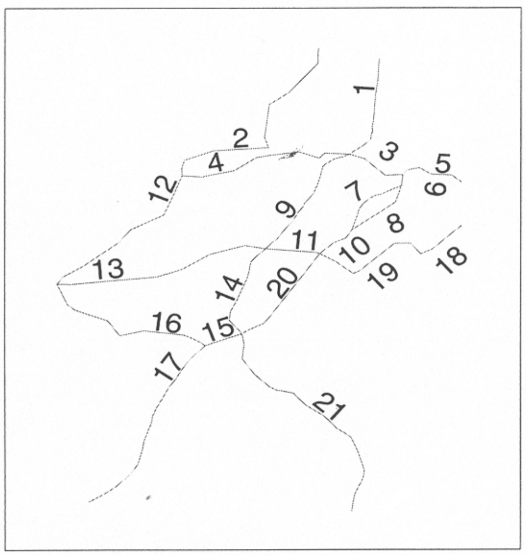


Figure 4.14 A partial highway network in northern New Jersey.

to a minimally connected network, a circuit emerges. Therefore, the number of circuits can be obtained by subtracting the number of edges required for a minimally connected network from the actual number of edges. That is, $e - (v - 1)$ or $e - v + 1$. In the network shown in Figure 4.13, with $e = 10$ and $v = 10$, the number of circuits is 1. This is formed by edges 4 and 5. For a given number of vertices, the maximum number of circuits is $2v - 5$. Therefore, with these two measures of a circuit, we can derive a ratio of the number of actual circuits to the number of maximum circuits. This ratio, which sometimes is known as the *Alpha Index*, is defined as

$$\alpha = \frac{e - v + 1}{2v - 5}.$$

Using the Alpha Index, we can compare the two networks in Figures 4.13 and 4.14. In the network in Maine, if not for the circuit formed by edges 4 and 5, this network would be a minimally connected network. Thus, the Alpha Index of this network is only 0.067. By contrast, the network in New Jersey is better connected, with several circuits identifiable through visual inspection. The Alpha Index turns out to be 0.172, more than twice as high as the index for the Maine network.

ArcView Notes In the project file `Ch4.apr`, a new function is added to the standard ArcView user interface to calculate the two network indices. The procedure first identifies the number of edges and the number of vertices in the network. ArcView, which does not utilize the arc-node topology in the data, as in ARC/INFO, does not offer simple methods to identify nodes or vertices in the network. The procedure basically decomposes all polylines into points in order to identify the endpoints of each polyline. By comparing the endpoints of different polylines, the procedure identifies the number of vertices. On the basis of the number of vertices and the number of edges, the two indices are derived.

4.4.2 Accessibility

The two indices discussed above assess the characteristics of the entire network. Different individual elements, either vertices or edges, have different characteristics or relationships throughout the network. Therefore, it is necessary to analyze the characteristics of each of these elements. In general, the procedure is to evaluate the accessibility of individual element in regard to the entire network.

Traditionally, network analysis literature uses vertices or nodes as the basic elements of analysis (Taaffe, Gauthier, and O'Kelly, 1996). Conceptually, using links or edges as the basic elements is also appropriate because the accessibility results are now applied to the edges but not the vertices. In the rest of this chapter, our discussion will focus on the accessibility of individual edges.

A simple analysis is to identify how many edges a given edge is directly connected to. If the edge is well connected, then many edges will be linked to it directly. For instance, in the network in Maine, link 1 is well connected because it has four direct links to other edges. Links 3 and 7 have the same level of accessibility. On the other hand, Links 2, 6, 8, 9 and 10 share the same level of accessibility based upon the number of direct links. To a large degree, this information is already captured in the connectivity matrix such as the one shown in Figure 4.13. The binary connectivity matrix shows a 1 if the corresponding edges are directly linked and 0 otherwise. Therefore, in order to find out how many direct links are established for an edge, we just need to add all the 1s across the columns for the corresponding rows. Table 4.5 shows the connectivity matrix with the sum of direct links for each edge. The results confirm our earlier analysis based upon the map in Figures 4.8 and 4.13.

However, the number of direct links to an edge may not accurately reflect the accessibility of an edge. An edge with a small number of direct links may still be quite accessible, depending on its relative (topological) *location* in the network. For instance, an edge may have a small number of direct links but may be centrally located in the network, so that it is still well connected to other edges. Edge 2 in Figure 4.13 is such a case. It has only two direct links (to Links 1 and 3), but it is centrally located. Thus, it is easy to travel from this edge to all other edges in the network as compared to Link 6, which is located at the end of the network. To capture the relative location of a given edge in the network, we need to find out how many links are required to reach the farthest part of the network from that edge.

In a simple network like the one in Maine, it is quite easy visually to derive the number of links or steps required to reach the farthest edges. Using the edges identified, for Link 1, the most accessible one according to the number of direct links, three links or steps are required to reach the farthest edge (Link 6). Link 2, one of those with the lowest number of direct links, also requires three steps to reach the farthest edges (Links 6, 8, and 9). Therefore, even if the two edges have very different numbers of direct links, their locations in the network are equally desirable. Table 4.6 shows the number of direct links and the number of steps or links required to reach the farthest part of the network. The two measures do not necessary yield the same conclusion because they evaluate different aspects of the edges with respect to other edges. Please note that 1 plus the highest number of links or steps required to reach the farthest edge of the entire network is also known as the *diameter* of the network. In Figure 4.13, the highest number of steps required to reach the farthest edge of the network is five, which is defined by Links 6, 8, and 9. Therefore, the diameter of the network is 6, i.e., six edges are required to link the farthest parts of the network.

TABLE 4.5 Connectivity of the Network in Maine and the Number of Direct LInks

ID	1	2	3	4	5	6	7	8	9	10	Sum Links
1	0	1	1	0	0	0	1	0	0	1	4
2	1	0	1	0	0	0	0	0	0	0	2
3	1	1	0	1	1	0	0	0	0	0	4
4	0	0	1	0	1	1	0	0	0	0	3
5	0	0	1	1	0	1	0	0	0	0	3
6	0	0	0	1	1	0	0	0	0	0	2
7	1	0	0	0	0	0	0	1	1	1	4
8	0	0	0	0	0	0	1	0	1	0	2
9	0	0	0	0	0	0	1	1	0	0	2
10	1	0	0	0	0	0	1	0	0	0	2

TABLE 4.6 Number of Direct Links, Steps Required to Reach the Farthest Part of the Network, and Total Number of Direct and Indirect Links for Each Edge

ID	Link	Steps	All_Links
1	4	3	15
2	2	3	19
3	4	3	16
4	3	4	21
5	3	4	21
6	2	5	28
7	4	4	18
8	2	5	25
9	2	5	25
10	2	4	20

An analysis based solely on the number of direct links is not the most reliable. An edge may not have many direct links, but because of its location, it may be reached from other edges indirectly and thus may be reasonably accessible. Therefore, besides direct linkages, our analysis should take indirect linkages into consideration. But obviously, a direct link is better than an indirect link; therefore, the two types of links should be treated differently. Indirect links also have different degrees of connectivity. For instance in the network in Maine, Links 1 and 8 are indirectly linked. Because Link 7 is between them, this indirect link is inferior to a direct link. Links 2 and 8 are also indirectly linked. But between them are Links 1 and 7. Therefore, two steps are required to join Links 2 and 8, one more step than the indirect link between Links 1 and 8. Thus, using the number of steps between links can indicate the quality of the link. If more steps are required, the indirect link is less desirable.

On the basis of this idea, we can derive the number of direct and indirect links that will require all edges to be joined together for each given edge, but the indirect links will be weighted by the number of steps or the degree of indirectness. Apparently, the larger the number of these total links, the less accessible is the edge. A small number of total links implies that the edge is well connected directly and indirectly to other edges. Table 4.6 also includes a column of all links for each edge. Link 6 requires the largest number of direct and indirect links in order to connect the entire network; thus, it may be regarded as the most inaccessible edge. This result conforms with the results of the number of direct links and the maximum number of steps required to reach the farthest part of the network. However, the total number of links is better in differentiating Link 6 from Links 8 and 9. The latter two edges have the same number of direct links and steps as Link 6, but in terms of the total number of links, Link 6 is a little less efficient than the other two edges. Similarly, the other two measures cannot distinguish the accessibility between Links 1 and 3, but the total number of links shows that Link 1 is slightly more accessible than Link 3.

ArcView Notes

Project file `Ch4.apr` includes two new menu items: **Direct Links** and **Steps and All Links.** The first new function adds the number of direct links for each polyline to the feature table. The second new function adds the other two measures of accessibility to the feature table in ArcView. These two measures are the number of links or steps required to reach the farthest part of the network and the total number of direct and indirect links. In deriving the number of direct links, the procedure is identical to the procedure creating the connectivity matrix. The additional steps consist of adding the number of direct links for each edge and inserting them in the feature table in ArcView instead of the connectivity matrix.

To derive the number of links or steps required to reach the entire network, the procedure identifies direct links of a link or a selected set of links in a recursive manner until all edges are exhausted. To derive the total number of direct and indirect links, indirect links are given weights according to the number of steps. If the indirect link involves three edges, it has a weight of 3. All these weights are added together to show the total number of direct and indirect links.

Because these measures of accessibility are added to the feature table, they can be shown by maps. Figure 4.15 shows three maps for the three measures for the road network in Maine. These maps are a very effective graphic representation of the accessibility analysis. They are very informative in showing the relative levels of accessibility for different edges in the network.

Please note that the accessibility algorithms implemented here are for demonstration purpose only. Using especially the last function on large networks (more than 100 segments) may require extensive processing time. Traditional network analysis relies heavy on the connectivity matrix. Using various matrix manipulations and algebra, many results described in this chapter can be derived. Most GIS including ArcView, however, do not support advanced mathematical formulation and computation; therefore, the matrix approach for network analysis was not implemented here. The approach developed here is a noncomputational GIS approach exploiting the characteristics and behavior of GIS objects.

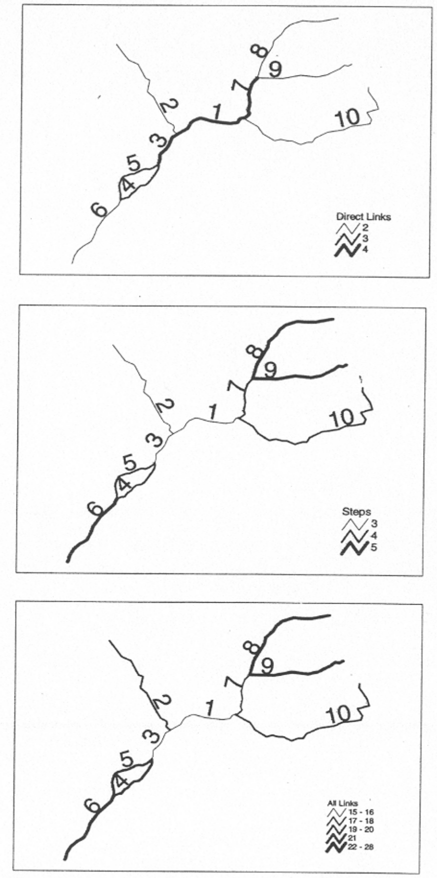


Figure 4.15 Maps showing the three measures of accessibility.

4.5 APPLICATION EXAMPLES

In this chapter, we have discussed several descriptive tools for analyzing linear features. These linear features may be completely or partially connected, such as fault lines or streams. The geometric characteristics analyzed in this chapter include their straight-line lengths, their network lengths, their overall directions or orientations, and consistency in their directions or orientations. These linear features can be fully connected, such as the road network of a given area or all tributaries of a river. Besides analyzing their geometric characteristics as if they are not fully connected, we can analyze how well they are connected to each other. We have discussed indices for the overall connectivity of the network and indices showing the relative accessibility of different network segments. In the balance of this chapter, we present two examples to demonstrate how the tools introduced in this chapter can be used.

4.5.1 Linear Feature Example

Using the drainage data for the northwestern corner of Loudoun County, Virginia, as an example, we will show how some of the tools we have discussed can be used to analyze a simple phenomenon. The terrain of northwestern Loudoun County is relatively rugged. On visual examination of the stream network, the drainage system appears to be divided by the mountain ridge running from north to south. Figure 4.16 shows the streams in that part of the county. It is clear that a long strip from north to south is clear of all streams and divides the system into two subsystems. If this assumption is correct and we have no additional geologic information, we may conclude that the streams on the two sides of the ridge probably have different directions.

Before we analyze the directions of the potential subsystems, we should analyze other geometric characteristics of the streams. Altogether there are 2,206 line segments in the network. Originally, there was no length information for the

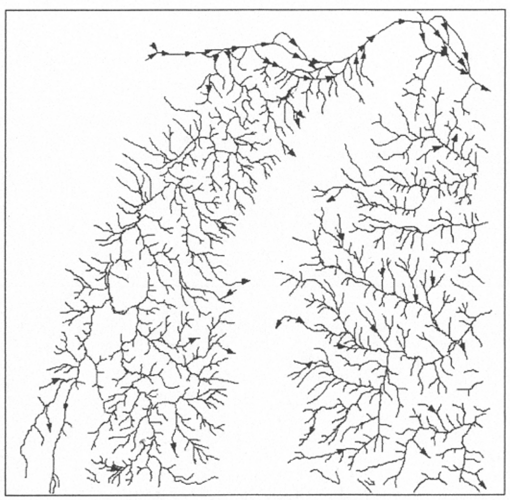


Figure 4.16 Selected streams in northwestern Loudoun County, Virginia.

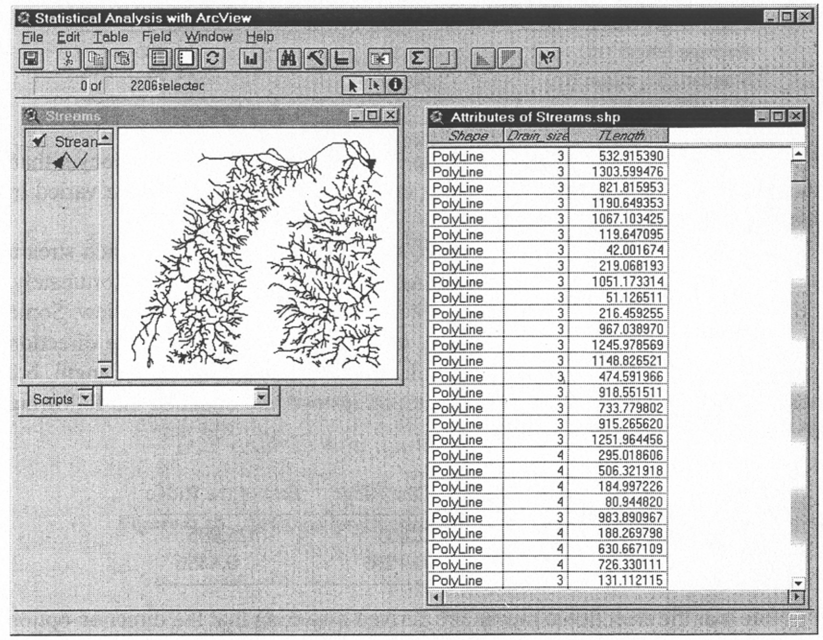


Figure 4.17 ArcView application window showing the true length of streams.

streams in the feature attribute table. Using the new function **Add Length and Angle** in the project file (`ch4.apr`) for this chapter, we can include the length of each stream in the feature table. We can also calculate the direction and orientation of each stream, but for this example, these measures will not be very useful. Figure 4.17 shows the application of ArcView with the feature table of the stream database. The table includes the true length of each stream segment.

To analyze the lengths of streams on the two sides of the ridge, we used the following procedure:

1. Using the selection tool in ArcView, select all streams on the west side of the ridge.
2. Open the feature attribute table of the streams, as shown in Figure 4.17, and highlight or select the field *TLength*.
3. Using the **Statistics** function in the table, derive the set of statistics.
4. Repeat the above steps for the east side of the ridge.

Below are the comparison of two statistics we found for the two sides of the ridge:

	West Side of the Ridge	East Side of the Ridge
Average length (m)	376.58	411.30
Standard deviation (m)	355.85	365.23

It seems that the streams on the east side of the ridge, on average, are longer than those on the west side, but the streams on the east side are also more varied in length.

In terms of direction of the streams, we could add an arrow to each stream segment using the new function **Add Arrow** in the project file. Unfortunately, because the streams are very dense, not all arrows appeared in the view. Some arrows did appear in Figure 4.16. Still, a more formal analysis of the direction is warranted. Using the procedure similar to the analysis of stream length, but selecting the **Directional Statistics** function instead, we obtained the following results:

	West of the Ridge	East of the Ridge
Directional mean	312.805	325.995
Circular variance	0.4536	0.4456

Note that the directional means are derived assuming that the direction option is selected. That is, the direction of flow of the stream is considered. Because both directional means are between 270 and 360 degrees, they point to the southeastern direction. Overall, streams on the east side of the ridge orient toward the east more than streams on the west side. Streams on both sides seem to vary in direction by a similar magnitude. But overall, we cannot say that streams on both sides of the ridge run in different directions.

One has to realize that main streams and tributaries run in different directions and sometimes are even orthogonal to each other. The above analysis did not distinguish streams of different sizes; therefore, the results may not be accurate. Using ArcView selection and query tools, we select larger streams (using the **Drain-size** attribute in the feature table to select **Drain-size 1** or **2** with 882 streams selected) from the entire region. The selected streams are shown in Figure 4.18. We performed the same analysis as above and achieved the following results:

	West of the Ridge	East of the Ridge
Directional mean	314.928	336.01
Circular variance	0.4147	0.38981

When the analysis is limited to larger streams, the directions on the two sides of the ridge diverge a bit more. A significant change occurs in the consistency of stream directions. The circular variances for both sides are smaller than before, and the decline in circular variance is very clear on the east side. These results indicate that streams on the east side flow more to the east, while streams on the west side flow more to the south-southeast.

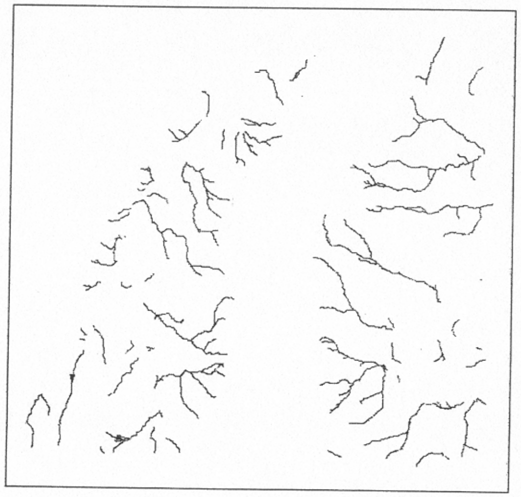


Figure 4.18 Selected larger streams, Loudoun County, Virginia.

4.5.2 Network Analysis Example

The other analytical tools introduced in this chapter are used for analyzing networks. Two data sets are selected as examples to demonstrate how these tools can be used. The Washington, DC, metropolitan area has one of the worst traffic conditions of any major U.S. cities. As the capital of the country, this city should have a very effective road network. Washington, DC, is surrounded by several local jurisdictions. Several Maryland counties are on the east and north, and several Virginia counties are on the west and south. The worst traffic areas are found in Montgomery County, Maryland, and several counties in northern Virginia, including Fairfax (including the independent city of Fairfax in the middle), Arlington, and the City of Alexandria. Figure 4.19 shows the boundaries of these jurisdictions, their road networks, and Washington, DC. The major road network data are divided into two groups: one for Montgomery County and the other for northern Virginia. The data set includes only major roads such as interstate highways and local major highways. Local streets are excluded for illustrative purposes. We will use the network analytical tools discussed in this chapter to analyze and compare these two road networks.

The following are the steps we took:

1. Move the Road Theme for northern Virginia to the top of the Table of Contents in the View window to ensure that the network analysis procedures will use that theme.
2. Under the **Analysis** menu, choose the menu item **Gamma_Alpha Indices**.

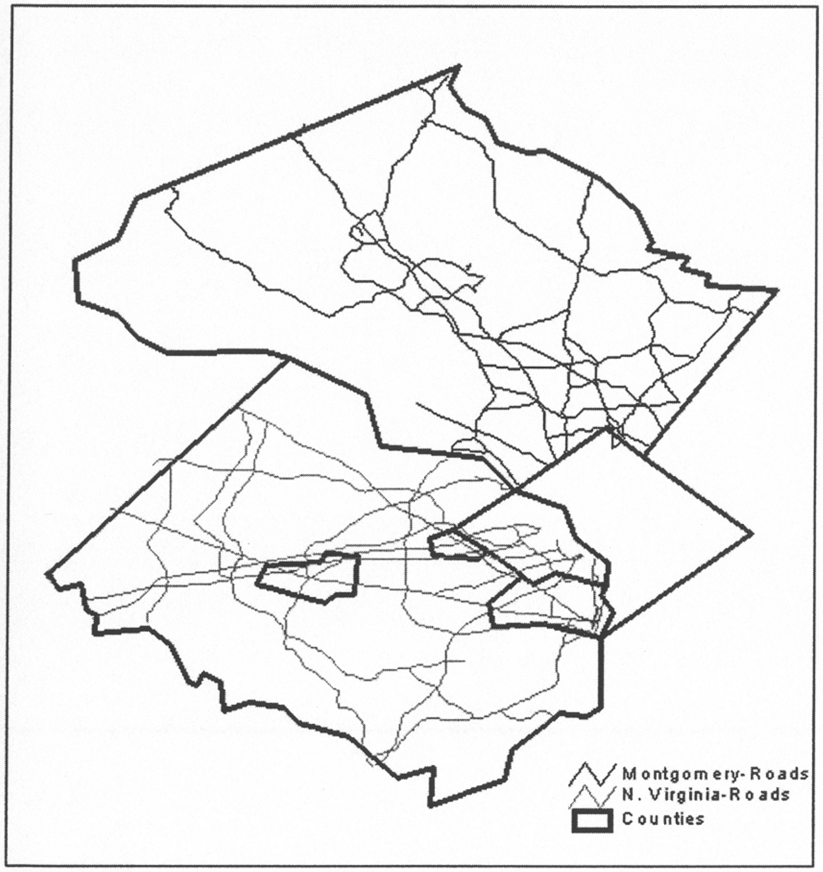


Figure 4.19 Selected road networks for northern Virginia and Montgomery County, Maryland.

3. If no feature is selected, ArcView will notify the user and ask the user if it is all right to use all features in the theme.
4. After clicking **OK**, a series of measures shows up in the window.
5. Move the Road Theme of Montgomery County to the top of the Table of Contents in the View window, and repeat steps 2 to 4 for Montgomery County.

The results from the two areas are reported in Table 4.7. It seems that northern Virginia has a denser major road network than Montgomery County, based upon the vertex numbers and numbers of edges. When the Gamma and Alpha indices are compared, it is true that northern Virginia scores higher on both, indicating that in terms of the number of edges or links and the number of circuits, the

TABLE 4.7 Gamma and Alpha Indices for Road Networks in Northern Virginia and Montgomery County, Maryland

	N. Virginia	Montgomery
No. of vertices	139	106
No. of edges	194	144
Gamma Index	0.472019	0.461538
Alpha Index	0.205128	0.188406

road network in northern Virginia is slightly better connected. But the differences between the two regions are quite small.

The Gamma and Alpha indices give us some general ideas about the overall connectivity of the two networks. It will be also interesting to see if different parts of the same network are equally accessible. In other words, we need to analyze the accessibility of the network locally, and the number of direct links for each road segment is a good indicator. Following the procedure for calculating the two indices, we choose the **Direct Links** menu item under the **Analysis** menu. This function calculates the direct links for each road segment. A larger number of links indicates better accessibility. After this menu item is selected, the number of direct links for each segment will be calculated and the result is stored as the new attribute link number in the attribute table. We conduct the same calculation for both regions. Because the numbers of direct links are now stored in the attribute table, these numbers can be used as an attribute for mapping. To do that, follow the steps below:

1. Double click the legend of the theme to invoke the **Legend Editor**.
2. In the **Legend Editor**, choose **Graduated Symbol** as the **Legend Type**.
3. In the **Classification Field**, choose **Link** as the field.

The width of a road segment is now proportional to the number of direct links of that road segment.

The results are shown in Figure 4.20. It is clear that road segments in both regions have many direct links. Closer visual inspection reveals that most segments in northern Virginia have multiple direct links, while certain road segments at the fringe of Montgomery County have relatively few direct links. Road segments with many direct links are limited to the so-called capital beltway and the Interstate 270 corridor. Road segments outside of these areas have fewer direct links. On the other hand, the direct links are quite evenly distributed in northern Virginia.

In our previous discussion, we included two other measures of accessibility: the number of steps required to reach the farthest edge of the network for each segment and the total number of direct and indirect links for each segment. These two measures are available by choosing the menu item **Steps** and **All Links** under

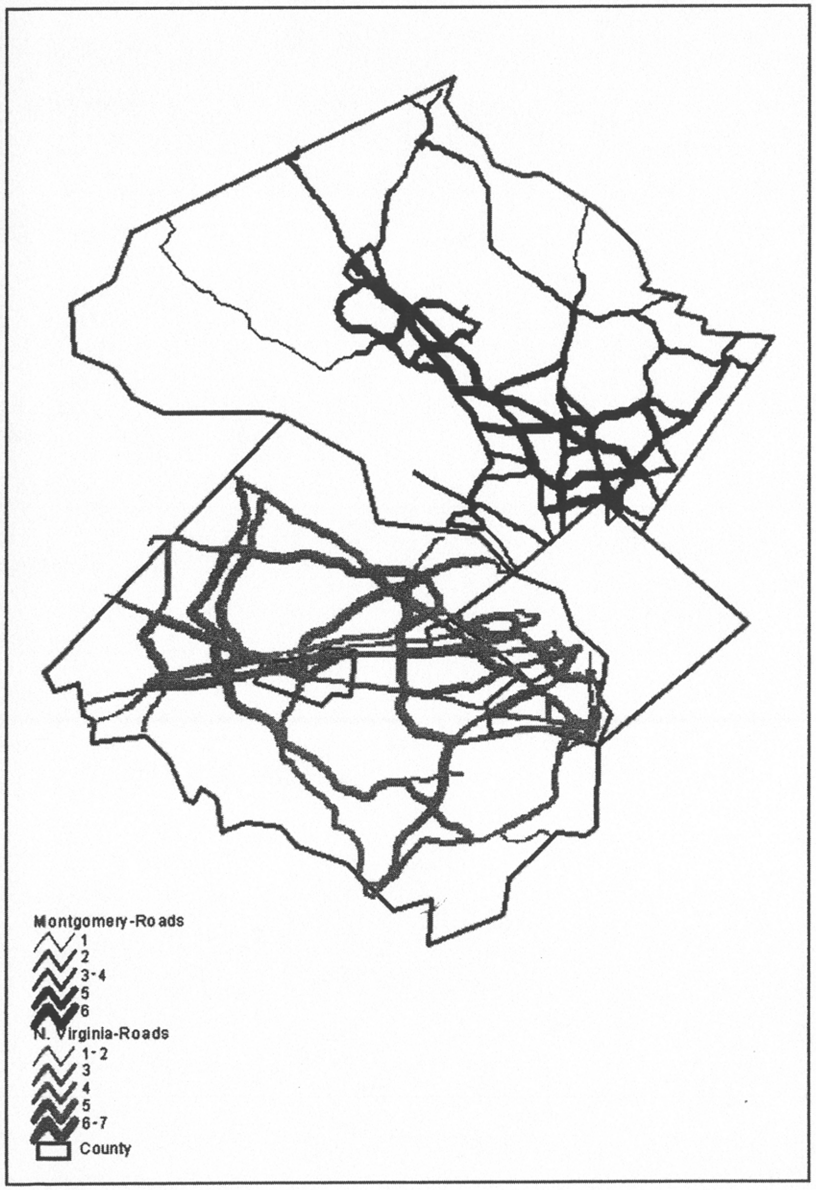


Figure 4.20 Number of direct links, selected northern Virginia and Montgomery County road networks.

the **Analysis** item. Repeating the set of steps we used before but choosing this menu item, the two measures for each road segment were added to their feature attribute tables. Then, modifying the legend via the **Legend Editor** and using **Graduate Symbol**, we created maps representing these two measures. They are shown in Figure 4.21 and 4.22.

In Figure 4.21, it seems that certain road segments in northern Virginia are less accessible than those in Montgomery County because those segments require 19 steps in order to reach the farthest edge of the network. This conclusion, however, is not valid because these two road systems have different sizes. The northern Vir-

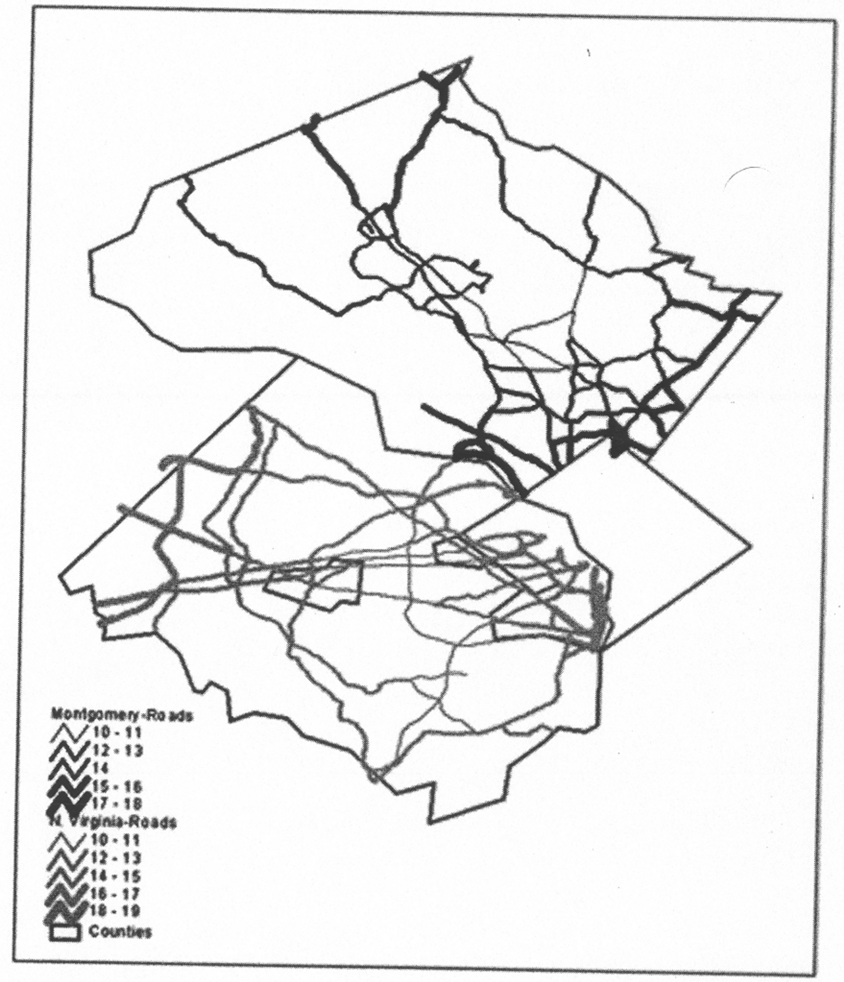


Figure 4.21 Numbers of steps reaching the entire network, selected northern Virginia and Montgomery County road networks.

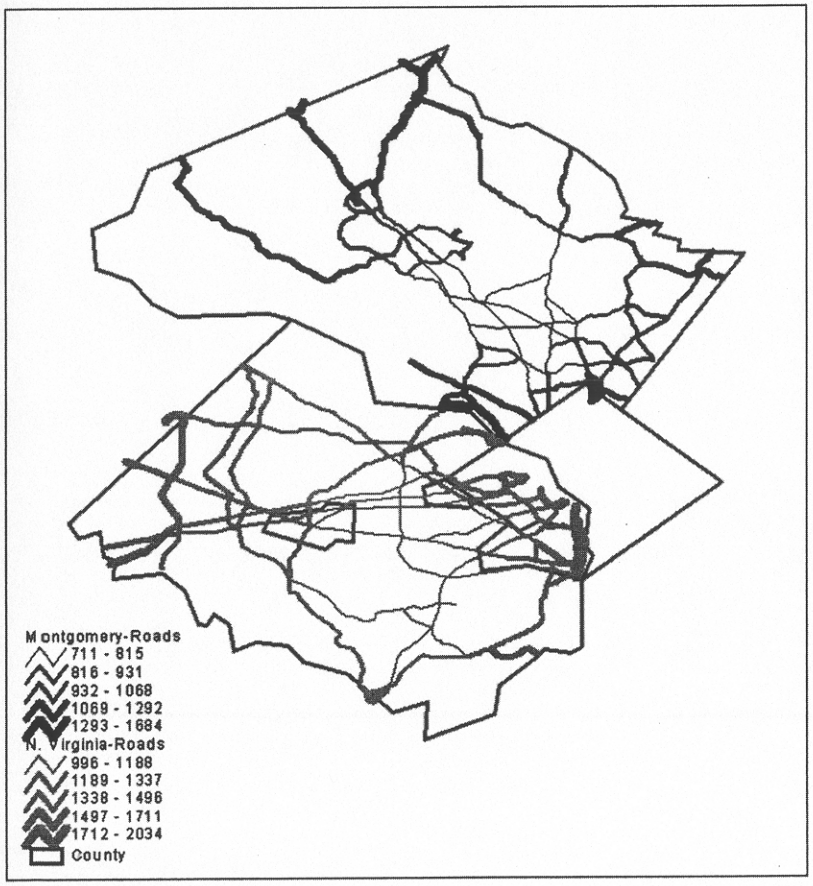


Figure 4.22 Total number of links (direct and indirect) in selected northern Virginia and Montgomery County road networks.

ginia system has more edges and nodes than the system in Montgomery County. Therefore, using this measure to compare the two areas is not appropriate. But within the same region, the number of steps is quite effective in indicating the relative accessibility of different locations. The central and southern parts of northern Virginia are highly accessible using this measure. In Montgomery County, the most accessible section is slightly north of the capital beltway in the southeastern section of the county.

Using the total number of direct and indirect links yields similar results. As mentioned before, we cannot compare the two regions on the basis of this measure because their networks have different sizes. On the basis of the total number of direct and indirect links alone, it seems that northern Virginia's system (highest 2034) is not very efficient because the number of links is much higher than that in Montgomery County (highest, 1,684), but northern Virginia's system is larger. Within the two regions, the results closely resemble those achieved using the number of steps. However, Figures 4.21 and 4.22 are very effective in showing the relative accessibility of road segments.

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