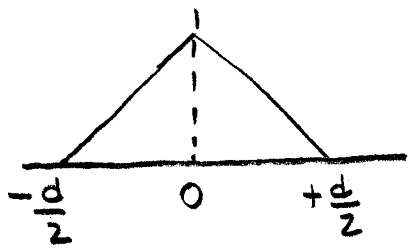
Ch 471, Fourier Transform + Convolution Problems

1. (a) $\int_{-\infty}^{+\infty} f(t) e^{-i\omega t} dt = \int_{-\frac{d}{2}}^{+\frac{d}{2}} e^{-i\omega t} dt = -\frac{1}{i\omega}\left[e^{-i\omega\frac{d}{2}} - e^{i\omega\frac{d}{2}}\right]$

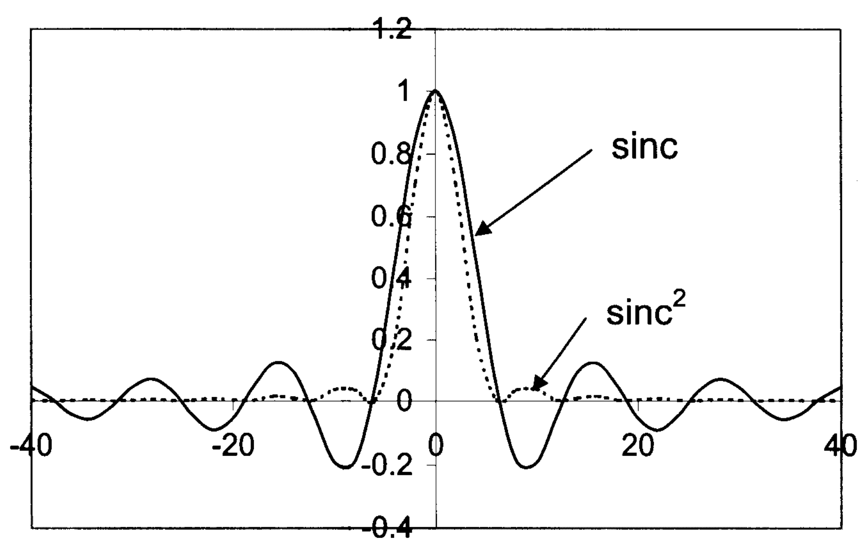
$= -\frac{1}{i\omega}\left\{\left[\cancel{\cos\frac{\omega d}{2}} - i\sin\frac{\omega d}{2}\right] - \left[\cancel{\cos\frac{\omega d}{2}} + i\sin\frac{\omega d}{2}\right]\right\}$

$= \frac{2}{\omega}\sin\frac{\omega d}{2} = d\,\frac{\sin(\omega d/2)}{(\omega d/2)}$ the "sinc" function

(b) "sinc^2" $= \frac{d}{2}\,\frac{\sin^2(\omega d/4)}{(\omega d/4)^2}$



(c) plots



2. (a) The sinc function corresponds to a non-apodized FTIR spectrum, while the sinc^2 is apodized

	Peaks	LOD	Resolution
boxcar	ringing sidebands	higher	better
triangular	less ringing	lower	worse

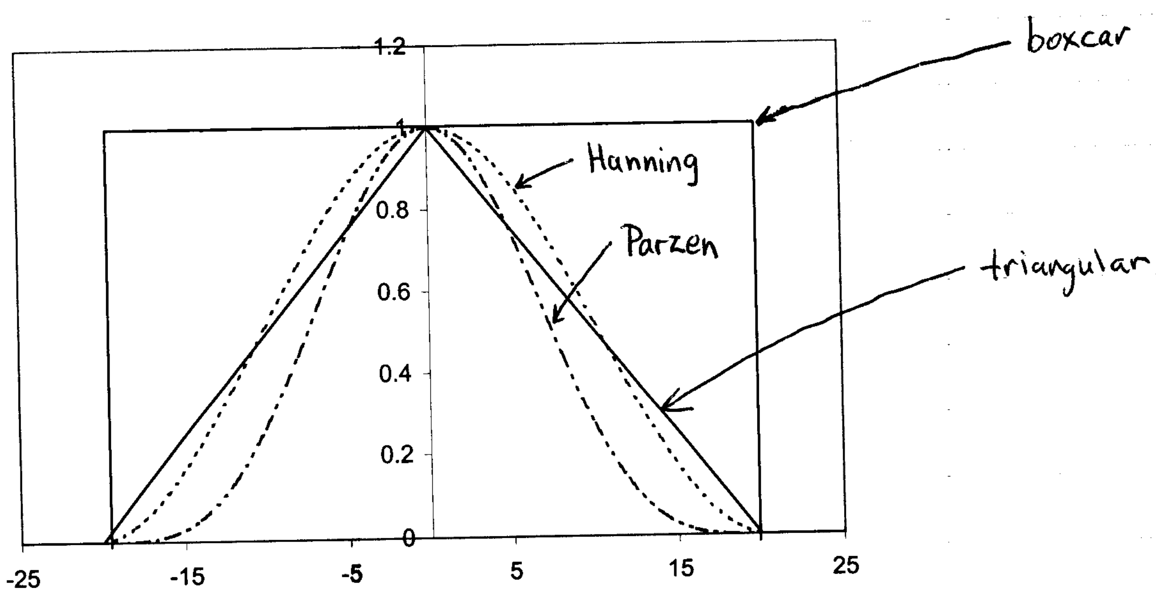
(b) $$\text{Hanning} = \frac{1}{2}\left[1 + \cos\frac{2\pi n}{N}\right]$$

$$\text{Parzen} = (\text{Hanning})^2$$

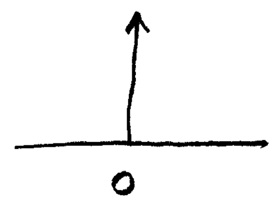
N = # sampled points in the FTIR spectrum

n = the n^{th} point

These apodization windows produce intermediate results between the boxcar and triangular windows.

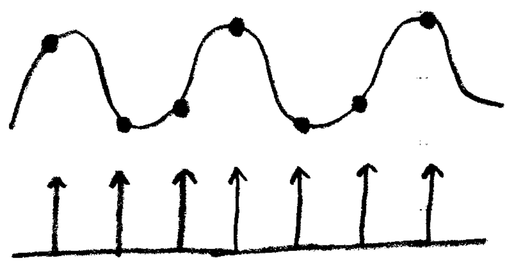


3. (a) $\delta(t) = \lim\limits_{\sigma \to \infty} \int_{-\infty}^{\infty} e^{-t/2\sigma^2}\, dt$



(b) $\mathcal{F}[\delta(t)] = 1$

(c) convolution with signal results in a series of sampled (digitized) data points, e.g.



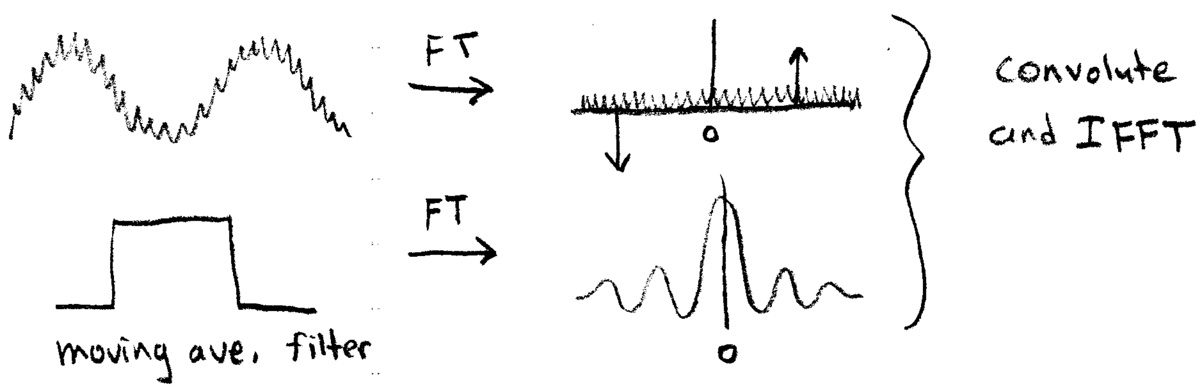
4. (a) $f_1(t) * f_2(t) = \mathcal{F}^{-1}[\mathcal{F}_1(\omega) \cdot \mathcal{F}_2(\omega)]$

$$2\pi f_1(t) \cdot f_2(t) = \mathcal{F}^{-1}[\mathcal{F}_1(\omega) * \mathcal{F}_2(\omega)]$$

(b) the 1st one

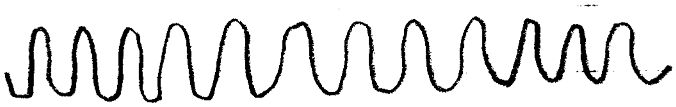
(c) the 2nd

(d)



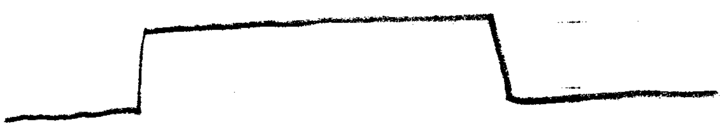
cont'd →

4 (d) cont'd



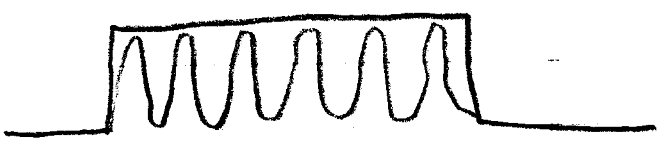
infinite cosine
(interferogram)

×



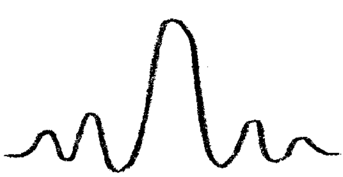
boxcar

=



Sampled interferogram

↓ FT



IR peak
(no apodization)